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CONTENTS

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Volume XXXIX Number 1

Edited by William David Reeve

The Effects of Military Training Upon General Education

By Henry W. Syer

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I. Introduction—A General View

Hundreds of thousands of words have been printed, in newspapers and magazines throughout the country, commenting, explaining and conjecturing as to what the returning veteran will want and demand. But relatively little thought has been given as to what he may be able to do for us in bringing new information, new ideas and new methods for performing old and familiar tasks.

Each veteran has undergone strict training under military methods, training far removed from the vague, impractical, bookish education still predominant in many of our educational institutions. Each has learned, in a minimum time, skills vital to the protection of his life and the destruction of the enemy. Will he be satisfied to sit quietly in a classroom, when he resumes his interrupted education, and have theories thrust at him, although practical and more interesting methods could be employed? Why not consider what Military Training Methods have to offer us, that we may be ready, not only to make practical the civil education of these men and women, but heighten the interest and more rapidly increase the skills of the younger students?

The author, while disclaiming any great authority on the subject, nevertheless considers himself fortunate to have been able to add more than three years of association with Army Training Methods to his pre-war experience teaching mathematics in one of the leading secondary schools of the country.

Let us then attempt to evaluate the methods used so extensively in Military Training and suggest a means by which they may be adapted to General Education.

II. The Army's Contributions to Education

The most conspicuous contribution is the development of visual and auditory methods. Films, film slides, phonograph records, large portfolios of colored diagrams, posters, models, sand tables, and demonstrations have been used in the army to maintain interest and teach many who would never have learned from the written and spoken word alone. This use of a new power, multi-sensory education, will be discussed at much greater length in section III of this article. It must shortly have its own place in the school system.

While many of these training aids were of great benefit to the student, since they insisted on his participating in, as well as

witnessing the demonstration, it must be admitted that seldom was his performance an actual experiment to discover the correct way to use a skill. More often it was a simple imitation of someone's idea of the correct procedure. With the additional time available in civilian schools this type of instruction could be improved and developed into laboratory courses where ideas are developed by handling material objects, not merely courses where the technique of manipulating tools and materials is taught. Here is the great difference between learning by performance within the army or outside the army. The Army demanded only that the student learn to perform the skill being taught in one most effective manner. General education requires that the learning of the correct method be followed by a generalization, then a realization of the place occupied by the principles involved in the pattern of all knowledge—or at least in the pattern of all education acquired up to that time.

Since the subjects taught in the army were concerned with a specific correct procedure instead of either a generalized correct procedure or the development of an attitude, it was easier to formulate the goal of this learning in terms of physical skills. Motivation then was simple, the objective often being seen in the demonstration. When any educational goal can be thus concretely formulated so will its motivation be easy.

Demonstrable goals, however, may at times have a harmful influence. Physical, immediate goals, are sufficiently attractive and concrete that schools will be criticized for trying to employ more idealized objectives. Students, remembering the clear-cut goals of military training will claim schools are impractical, idealistic, time wasting, unworldly—and therefore useless. We must keep in mind, however, that though goals in terms of practical, physical ends should be used; whenever possible, other goals, less tangible, must be expressed and sought for.

There is one other, possibly harmful, influence caused by the accelerated type of training used by the Army. Time was too short to lead instructors through the theories of teaching or even to be sure those most interested in teaching were selected. The problem was to show the instructors a fast and certain method of presenting material which would assure a maximum amount of learning in the shortest possible time. A mechanical, stereotyped lesson and teaching plan was developed which could be passed down from a training director far removed from the scene of the teaching. This plan included a synopsis of material to use together with objectives and methods of presentation. Little opportunity was given the instructor to use his personality or let his interest in the subject appear. Such frozen fact lessons would have been impossible were the subject matter not so material and concrete. Many courses, where ideas are more important than facts, would be ruined by such overpowering direction. However, the positive contributions of military training far outweigh the harmful ones. We must be reminded that we are here judging the effects of military training upon the future of the educational system, not upon the individuals who survived it.

The use, however, of standardized teaching units for subjects which have as their fundamental objectives the learning of a basic core of facts could be well worthwhile. If well defined units in spelling, multiplication tables, history dates, lists of important works by authors studied in literature, valences in chemistry, verb forms and vocabularies in foreign language studies, and similar factual units were available the individual classroom teacher could be relieved of a large part of the now necessary planning and teaching. It might even be possible to organize these subjects into self teaching units which the student could study and restudy, testing himself all the while, until he reached the level of proficiency required.

Education is most justly administered when all students are measured against

the same standards. With this idea in mind the Army has successfully developed complete sets of tests for all important types of subjects taught. These have been carefully compiled, fully pretested, completely standardized and regularly revised. Standards have been devised for industrial skills, job aptitude, academic aptitude, achievement in elementary school subjects, classification based upon general intelligence, general information, mechanical aptitude. In addition check lists were constructed to aid in judging proficiency and experience in former occupations, and aids to neuropsychiatric screening. Similar tests in many of these categories would be very useful in school systems though unavailable because of the expense of compiling. The Army was justified since expenditure per capita drops astonishingly when large groups are involved.

City high schools, colleges, rural educational areas or state-wide groups centered in university departments of education could profitably use these army tests and adapt them, in the light of knowledge of their past use, to the needs of the educational system of the country. Though the Army has compiled the material and borne the expense, the tests should neither be discarded, now the war is over, nor be entirely monopolized by the Army.

The Army has furthermore attempted to define the elusive qualities of leadership in their non-commissioned officer and officer candidate schools and has arrived at standards and methods of developing leadership far superior to any method civilian education has supplied in the past. Even though leadership in peace may differ from leadership on the battlefield a program to train leaders for industry and government could profitably grow from the beginnings the Army has made.

Furthermore army methods have produced remarkable results in teaching illiterate soldiers. It is clear then, from the rapid progress these individuals make in reading, writing, and arithmetic, that the illiterates of our country are not uneducable, but have merely lacked the opportunity, the incentive, and the proper methods. Educators would find most worthwhile a study of the books and method used in army training centers to instruct these people rapidly in the fundamental tools of education.

III. Plan for a Department of Teaching Aids

In discussing more fully the ideas previously mentioned most important in our opinion is the development of teaching aids. Though termed Training Aids by the Army we believe Teaching Aids to be a better term, since it suggests a greater application to all types of learning. *Training* has taken on the connotation of physical training, or training for skills in physical manipulation. This same distinction has been made in naming *vocational training* rather than *vocational teaching* departments of schools. Academic types of education sometimes require minds able to think in terms of abstract symbols without the aid of pictures and objects. It has been assumed, therefore, that the more academic subjects should be taught from books and the less abstract subjects by demonstrations, models, laboratories and shops. But by adding teaching aids to the usual methods of teaching such subjects as mathematics, English, Foreign Language, history, and geography, the progress of the average pupil would be much faster and the pupil who formerly found these subjects impossible would find his discouragement lifting. Pictures and models mean more to all people than words. In fact, for some people words alone convey no true or direct concept of the idea under discussion. For very abstract ideas the model or analogy used may resemble the basic idea in perhaps only one or two aspects. In such cases words alone will give a true description of the abstraction. For instance, no single example will give the real meaning of "truth," "beauty," "lightness," "distrust," "sufficiency," "preparedness," "horror," or any other ab-

stract idea. Similarly, no single teaching aid will illustrate such terms as "velocity," "mass," "rhythm," or "justice." The meaning of these words is realized only by inductively abstracting their essence from repeated, different examples. But in almost all secondary school education, and in a large part of college teaching the ideas are concrete enough so that visual, auditory and other sensory aids will be of great help.

At present only the physical sciences have developed teaching aids to any worthwhile extent. Unfortunately, however, their development was not a growth from interest in improving teaching methods, but arose naturally from the subject matter itself. Physics, chemistry, and biology profess so constantly to be studying the world around us that any course teaching these sciences without handling physical materials would certainly invite criticism. So the use of the teaching aids which the physical sciences have developed is still incorrect, for their attitude is: "We should teach science *and* let them experiment in the laboratory," instead of "We should teach science *by* letting them experiment in the laboratory." (See part IV of this report: A Mathematics Laboratory Course). Science has made better use of teaching aids other than laboratory work, such as films, film strips, models, museums, field trips, and demonstrations. These have been organized as the need for them arose. On the other hand, laboratory work has been planned to fill just so many hours a week, regardless of the real need for any particular week.

Why is it more departments have not spread out and embraced new, useful teaching aids as they appear? Chiefly there are two reasons. Teachers do not hear about these ideas until they have been well developed, and there is not sufficient time during a teacher's day to track down, investigate, develop and utilize all new ideas which do appear.

Consider the subject of educational films alone. The classroom teacher does not have time to read all the information appearing about films which apply to his own subject, see showings of these films to evaluate which could be used during the year and decide what place they would have in the course. Furthermore, he has not time to sell the idea of buying sufficient projection equipment to the school board or the treasurer of the school, order the film on time, train someone to run the projector and finally wrap up the film and send it back to the rental agency. Even when the film is the property of the school, its use and care still add to the day's work.

If in addition of all this he is told he should make colorful charts describing pictorial methods of teaching his subject, should maintain a file of pictures and photographs to illustrate classroom work, should develop practical exercises to make learning interesting, should construct models to show how his subject applies to the world about him, should keep in touch with the latest books published in his own field, should devise objective-type tests and standardize them over a period of years, should be ready at a moment's notice to advise the purchasing section of new materials needed and the vendor's name and prices (just in case the budget allows a purchase), should arrange regular field trips and visits to industrial plants, should be on the lookout for any new textbooks on his subject, and should *above all* save plenty of time for individual help for the problems in learning each of his pupils will have, then, certainly, no teacher can be blamed for giving up and just teaching out of the book so many pages a day. Industry and the Army have found their sub-divisions need outside aid to teach in the most efficient manner—why not the schools?

The answer to this difficulty is the formation of a separate Department of Teaching Aids. The location of this department, its work, requirements and administration will be discussed in the next few paragraphs.

Naturally, a Department of Teaching

Aids may be as simple or as complex as the schools wish to make it. To develop the idea fully the following description assumes a large, complete department will be established aimed at giving the greatest possible amount of aid to the rest of the school. Even a partial realization, however, of these ideas would be better than none at all.

Where would such a department be located? For a large city high school, a large private school, a college or university, the department would be part of the school and parallel the other departments in the organization. Small high schools, elementary schools, and small private schools could band together to support a department for the whole group. The most natural grouping would be by geographical location. All schools in a township, county, or even in a state could be supplied by one department of teaching aids. It should be stressed, however, that the nearer the department be to the classroom teacher, the better it can perform its task. Schools in a district around a state university would logically find a focal point there for the department. Great care must be taken, however, to make sure the department serves the needs of the teachers and does not try to use the schools in its territory as experimental schools for university-bred ideas. In *every* instance, the department of teaching aids should be the servant of the other departments, and not the source of teaching policy.

What would be the duties of the personnel in the teaching aids department? They would keep up-to-date bibliographies of books on teaching methods, magazine articles from professional and general publications, lists of available films, film strips, pictures and their sources, and would publish monthly lists of this information to be circulated to the departments and then kept in a permanent file. They would help choose text books for new courses, or to replace out of date books in regular courses, and would even stand ready to write books as needed or edit and supervise the reproduction of books written for their courses by members of the faculty. They would maintain a file of pictures, films and film strips to be delivered to classrooms as required by the teaching schedules (books and magazines would still be kept in the school library). They would collect pictures, make posters and charts, and be prepared to make still photographs and motion pictures, both in black and white and in color, as required to improve the teaching in the schools. Among these workers would be included trained artisans who could work in wood and metal to produce models and demonstration equipment as needed by various departments and who are experienced in electrical wiring, relays and motors, having a knowledge of how to work such varied materials as plastics, cloth and string. Therefore they could produce working models which could become a permanent part of the school's teaching collection. They would devise and standardize tests for all units within courses and overall tests for complete courses, whenever such tests would be helpful. They would have catalogs of all commercially available teaching aids and stand ready to advise the departments as to the best sources of material. They would serve as a clearing house for all exhibits, assembly programs, extracurricular club activities, school museums, and visits by groups to industrial plants, concerts, athletic events, and theater parties which would make of them experts on transportation problems and public relations contacts as effecting the classroom teacher. All the above are tasks which the individual teacher is expected to do for himself, but from which he would be relieved by the creation of such a department. Does not this seem to be a sufficient justification?

How should such a department be administered? Definitely the actual work should be done by full time, experienced workers who have no other position than that which they hold in the teaching aids

department. They should be hired for the very special skills required of them, for their interest in education and in experimenting with new ideas. The supervisor should be a teacher who continues to teach regular classes as part of his duty. Aiding him should be a committee representing all departments, which meets regularly to discuss budget allotments, priorities of purchases and construction of teaching aids. Obviously, if the Physics department wants an expensive wind tunnel and the History department wants a series of motion pictures purchased, both expensive projects, someone must decide which comes first.

How many employees would be required to carry on the work outlined? This depends upon the number of pupils served. For 500 to 1,000 pupils, two employees should be sufficient. Number one could maintain the bibliographies, do the writing and editing, be responsible for standardized tests, serve as a purchasing counselor, and arrange outside visits. Number two could make the charts, posters, and mechanical teaching aids as well as the still and motion picture photography. For 1,000 to 2,500 pupils three employees would be required, the work of number two above being divided so that number two would do the art work and photography and number three the mechanical models and demonstration equipment. For 2,500 to 5,000 pupils four employees should be sufficient. Number one in this case would do the bibliographies, editorial work, purchase counseling, and trip planning; number two would do the art work and photography; number three the standardized test program; and number four the mechanical teaching aids and demonstration equipment. If 5,000 to 10,000 pupils were being served five employees and a full time director would be needed. Here the separation of duties could be as above with purchase counseling and trip planning taken from number one and performed by number five. It would be impractical for a single teaching aids department to serve more than 10,000 pupils.

How would teaching aids be chosen for purchase or construction? Naturally by the teachers themselves. One of the chief objections to a centralized bureau is that it will take initiative and spirit out of each teacher's work. True, it might have the same effect as if some higher headquarters told him exactly how to teach. To avoid this, and to prevent each teacher from getting a roomful of teaching aids with which he does not sympathize, and which therefore he does not use, teaching aids would be supplied only upon the request from the individual teacher. This request could be a detailed plan with diagrams, sketches and specifications for construction, or it could be merely a simple request to: "develop a teaching aid to make it easier to learn the capitals of the states." Other types of requests might be: "How can I demonstrate carburation in gasoline engines?," "What films can we buy to use in a course on costume design?," "Will the teaching aids department please make a moving picture illustrating the development of the state of ____________ (wherever the school system is located)," "May the biology classes borrow the equipment to make a set of colored still pictures of a group of birds who are building a nest outside our window?," "Can a test be developed to measure proficiency in cooking?", "Would it be possible to take a group of fifty to Chicago next month to the symphony concert, and how much would it cost?," "Can you make a chart showing the influences of foreign groups upon the development of customs in the U.S.A.? This should be accompanied by film strips, lantern slides, or mounted pictures to illustrate the unit.", "Would it be possible to make a graph to prove to the school committee that we are teaching at a lower rate per capita than ever before? They want to cut our budget." Naturally, most requests would be more detailed than these listed here, but this should give some idea as to the work covered by the teaching aids department. After a request is received, it

is clarified by telephone calls and visits with the teacher or department from which it originated until the teaching aids department is sure it knows just what results are desired. In some cases changes may be suggested or the idea shown to be impossible to develop. If the request is simple or routine it is put into production at once; otherwise, it is held for the next committee meeting to decide whether funds and time can be allowed for it. When requests are put into production which easily admit of mass production (such as printing, silkscreen posters, film strips, movies, or simply-made models) then descriptions will be circulated to all interested teachers to see if they wish to submit a request for a copy. Notice that in every case the initiative and decision comes from the classroom teacher.

This set of suggestions and directions for organizing a department of teaching aids is based upon experience with the army's production of training aids and a study of their faults and virtues. The one goal of such a department is to make classroom teaching easier and more efficient in the manner in which each teacher wishes it to develop. It must be a servant of the teacher.

IV. A Mathematics Laboratory Course

A further lesson to be learned from military training is the necessity for the learner to participate in the process or act which he is learning. The rifleman handles a gun (without ammunition) almost the first day. The operator of an antiaircraft director learns the "feel" of the controls immediately. Army administration is taught by filling out forms; telephone installation by installing phones; radio theory by building radios, first simple types and then more complicated ones; pilots "fly" early in a "Link Trainer"; and all soldiers are conditioned to battle noises by proceeding through an Infiltration Course area with live ammunition whizzing over their heads and small land mines exploding close to them. All these are examples of learning by doing.

One feature, though missing in army training, need be added in order to incorporate the method into the present system. Though not encouraged nor led into mistakes, the pupil must, however, be permitted to make initial mistakes if he does so naturally. Trial and error methods are too valuable to ignore. Speed was more important than reasoned learning in military education so there was not time to allow for mistakes. Performances were required to be correct the first time and this particular method considered to be the only right procedure. In leisurely peace time teaching, however, more exploration should be allowed and less complete direction supplied. Then, when the best method is finally chosen, each pupil will feel he himself has chosen it, and will remember it longer. Also there will be less wondering as to what might have happened had it not been done the one, best way. The student has already tried the many other ways and has reached his own conclusion. Not only will the best method have been chosen, but in addition, the method of discovery will have been learned.

Rather than show how all courses can be taught in an exploratory manner, let us design and exhibit a course for the teaching of mathematics by this laboratory, roll-up-your-sleeves-and-do-it method. There is no day by day curriculum to display but the objectives, activities, administration, and physical equipment needed for such a course will be discussed.

One of the objectives of a laboratory course is to provide knowledge and materials which will remain a part of the school and which can be used by future classes. As opposed to this, in the physics laboratory, for example, equipment has often been merely dusted off, put through its paces, and returned to the shelf until needed again. The pupil in such a case, has had no part in the development of the apparatus. It is simply available to him as his toys were during childhood. There-

fore, another of the objectives in the Mathematics course is the building of machines and models, which illustrate mathematical principles and which could be added to the school mathematical collection. In order to do this, there must be models listed which have never been constructed before, and which, when built, will be substantial enough to serve for years without replacement. If a pupil constructs models, made many times before, or which are already in the mathematics museum, he will be filled with no more enthusiasm than if he were to perform a monotonous physics experiment, a trite, routine mathematics problem, or solve an antique theorem from a book. The essence of a laboratory is doing something new—not merely repeating something already done. This objective brings life into laboratory work far more surely than simply handling physical objects in place of reading about them in a book. A laboratory can be as dull and academic as a book unless it leads to real discoveries.

Now, separating the two objectives bound together in the preceding discussion, we find first that a laboratory course must give the pupil a sense of contribution to the school, and second, it must be a place where real, new discoveries are being made. Otherwise, it is just another hurdle in the race called: "Getting an Education." Both of these objectives can be reached by making laboratory exercises contain, at least in part, new, untried experiments and constructions.

Too many laboratory courses teach such subjects as the Pythagorean theorem or the summation of an arithmetic series out of a book using the laboratory later only to show how these results can be applied to mechanical drawing. A better way would be to design laboratory exercises leading up to the Pythagorean theorem and the formula for the summation of arithmetic series. The facts of Euler's theorem connecting the vertices, edges, and faces of polyhedra or the earlier theorem establishing the maximum number of regular polyhedra need never be proved in class if they are first discovered in the laboratory.

Now that we have an idea of what we hope to accomplish by this laboratory course, what types of activities are available leading to these ends? There are four: drafting exercises, paper and model constructions, laboratory experiments, and field exercises.

The drafting exercises would naturally tie in best with facts from plane geometry. Possible exercises include ruler and compass constructions, exercises in perspective, mechanical drawing, construction of artistic geometric figures, geometric progressions of regular figures, constructions of the conic sections as intersections of families of lines, and the discovery of properties of geometric figures by drawing. Here are some examples of geometric facts which can be discovered through drawings: answers to locus problems, angle measurement in circles, many of the simpler theorems from projective geometry (Desargues' Theorem, Pascal's and Brianchon's Theorems, etc.). These do not need to be proved or even named, the facts being simple and clear. Many ideas might be introduced from modern geometry: the nine-point circle, the Theorem of Pappus, Ceva's Theorem, Ptolemy's Theorem, and other similar discoveries. No proofs, remember, just discoveries. In this group of drafting exercises might appear many applications of algebraic formulas, such as the formula for the square of a binomial, or a trinomial, or the formula for the summation of an arithmetic series, and the form of the nth term. In such discoveries, cross section paper would be very helpful.

The same type of work can be carried on to discover the facts of solid geometry. Cardboard, balsa wood, and plastic models can be made which lead to generalizations or acceptance of fundamental geometric facts. Some typical models might be made of various types of polyhedra; string models of surfaces; mechanical devices to show

construction of plane curves; plaster, wooden, or metal models of solid geometric forms; mathematical measuring and computing devices (planimeters, circular slide rules, etc.); linkages illustrating geometric principles; three dimensional models of complex algebraic curves; paper folding exercises; stitching exercises; and models which show mathematical principles in the world around us. It is from this group of activities that most of the permanent exhibitions of the mathematical museum will come. The more you work with it, the more ideas will develop. Many interested in photography will want to take a series of pictures formed around some mathematical principle or idea; some interested in writing will want to compile a considerable amount of information on a single mathematical subject to be made part of the laboratory collection. Let them. Such activities are proper if combined with more active manipulation of mathematical equipment.

The next type of laboratory activity, laboratory experiments, will require a great deal of development. It is readily acknowledged that science uses mathematics in many of its experiments, but the science laboratory treats mathematics as a type of handmaiden who should work without either asking questions or having any personality of her own. It should be the aim of the mathematics laboratory to formulate experiments with equipment from the physics, chemistry, biology, astronomy, economics, and military science departments which illustrate mathematical principles. These experiments can be grouped and performed in sequences. For example: let us say the subject is the trigonometric functions. The oscilloscope, apparatus to produce Lissajous figures, a harmonic analyzer, pendulums combined with stop watches, circular objects revolving so as to produce shadows, and the Jolly balance might all be used to study the mathematical principles of this group of functions. It is true that a completely new book (if a book were used) would need to be written . . . but why not? Actually, as we will show later, a book is not at all necessary. In more elementary classes scales can be used to investigate the properties of equations, micrometers can be used to demonstrate the need for significant places in physical measurements, mirrors to study the laws of reflection, and glass jars filled with water to clarify the laws of refraction. Microscopes lead to the subject of proportion and speedometers to the conversion of units. The number of laboratory experiments will further grow from the suggestions of the students themselves once the course is well under way.

The most advanced and interesting type of laboratory work is the field exercise. The first exercises utilized here would be the usual ones of measuring heights and inaccessible distances. Transits and stadia rods should be available and the discoveries resulting from their use could proceed from simple facts about similar figures to more complex facts about trigonometry. Exercises in surveying, map making, and navigation of more and more advanced difficulty could be used. The scope at this point is limited only by time, expense and the experience of the teacher. For, although it may not seem so, there is a teacher behind all this effort, guiding the work, making suggestions, and vetoing wild ideas. The laboratory course must never at any stage turn into a free-for-all, with every one dabbling into the work that has the most glamorous apparatus.

Now for the discussion of the actual administration of the course, as it would appear in the school curriculum.

The laboratory course should be available in the curriculum at two levels; the first corresponding roughly to the tenth and eleventh years of school and the second to the twelfth year. The dividing line should be about as follows: those having only one year of algebra and no geometry would fit into the first course; those who have had plane geometry and at least one year of algebra would fit into the second. The work should be flexible enough so

that these prerequisites are not absolutely necessary, but they give the administrator some foundation for placing pupils in the courses.

There is not the slightest doubt but that everyone who studies mathematics could profit by learning part of it by the laboratory method. It is far too early to assert that it is the only way to teach this subject at the secondary school level; time alone will determine that. Although it is true that the more academic minded, college bound students should get the benefit of new courses, it is traditional to try them out on those pupils who find mathematics difficult. This is an unfair test. When the less facile pupils learn little, the conservatives shake their heads and say: "I told you that new method was no good." The truth is that the pupils tested would have learned even less from the usual course and never very much from any course. It is not wise at this stage to arbitrarily state that one certain group should study laboratory courses and others should study in traditional book-recitation courses. Both types, of course, have their places.

If the school is large enough one group of slower pupils and one group of faster pupils should be given instruction under each of two classifications: one course for those who do not expect to study Mathematics further, and the other for those who do plan to continue Mathematics. By observing the progress of these groups many questions will be answered. Here are some of them. Will laboratory courses serve the interests of slow and fast learners alike? What will the effect be on the mathematically minded as well as on the humanists and philosophers? What foundation for future Mathematics does a laboratory course provide? Where is there a place in the laboratory type course to teach the methods of formal proof? How much more difficult will it be to teach a laboratory course in private schools where the pupils do not have their homes and neighborhoods from which to draw material? In which type of school is such a course more important? These and many more questions will be asked and answered, at least in part, with the formation and development of such a course.

Five or six periods a week of fifty minutes or one hour length would be sufficient to provide continuity. They should be arranged in a 1–2–2 or 2–2–2 period grouping. The double periods will be the only ones permitting work to be set up and finished. A single period could be used only for explanation, group orientation, reading up on subjects to be studied, or tests.

Little group instruction will be possible in such a course. The teacher must only stand by to give advice and help. Though this may sound like a simple task it is many times more difficult to keep work planned for so many machines running down different tracks at different rates and to have some way to check their progress. In order to succeed most of the directions must be written up fully on mimeographed sheets, and most of the tests should be objective, self tests. Observation of the student at work will grow to be a larger and larger part of the evaluation.

For the early drawing exercises very explicit directions must be drawn up and published. As that work continues, less and less complete directions need be included on the written sheet. Toward the latter part of the work pupils could be told simply to look up a good idea in one of the geometry books on the shelf; write up their own directions and draw it. The groups preparing to work on model making would receive a few introductory lectures and discussions about the choice and manipulation of materials and the purpose behind the construction of their models . . . what they are supposed to illustrate. Former models would then be shown, with the explanation that they are now part of the permanent collection of models at the school. Following this introductory period, one or two models could

be constructed by the whole group after which each student would be on his own. Some money should be available in the budget to purchase materials hard to obtain, but not too much money. Half of the fun and profit is in acquiring the material, but if this search is too rugged many will lose interest. Students must be forced to plan ahead by submitting requests for future models. In addition, a card file of models that are needed for the collection could be maintained and the student allowed to choose his or her model from this file.

For those who are performing laboratory experiments with apparatus, it will first be necessary to instruct them in the handling of equipment and the responsibility it involves. A sample laboratory sheet would be valuable for discussion by the entire group, after which each pupil would perform a simple experiment by following directions supplied on carefully prepared laboratory sheets. Then they are on their own. For this work it is important to have all the laboratory equipment, or practically all of it, in the possession of the Mathematics laboratory. It should not be borrowed from other laboratories. First, this gives the Mathematics laboratory an importance of its own, taking it out of the poor relative class, less fortunate than the rich cousins—Physics and Chemistry. And second, with the pupils planning their own work to a great extent, it is necessary to have the equipment at hand when it is needed, without the inconvenience of borrowing it from some other section.

In the physics laboratory it is necessary to have enough micrometers on hand to supply an instrument for each individual. But the mathematics laboratory, where measurement is just as essential and fundamental, usually possesses only one or two, regardless of the size of the class. Why this difference? Because the pupil in the mathematics laboratory is required to plan his work so as not to conflict with others and consequently different pupils use the same apparatus at different times. To aid in this, items of equipment could be posted and signed for weeks in advance.

Field exercises would be performed under direct supervision at first, next by following printed directions, and later by using material referred to in books by page and number. As the student grows in ability exercises could be assigned by subject and objective alone, requiring him to locate books describing the correct method. Finally, verbal suggestions from the instructor would suffice. If at the end of the course a pupil, looking for a problem, approached his teacher and was asked to map the area between two nearby roads, showing by contour lines the shape of the hill between them, and if he, having never studied contour lines, started without further question to search for the definitions and methods necessary—any teacher would feel more than repaid for the past effort expended. As the course becomes more advanced, more and more dependence is placed upon vague suggestions which are followed to completion with the aid of a very complete library of books on *practical* mathematics. These books should be in the same room and as easily available as a pencil sharpener or similar tool.

A bit more must be said about the laboratory descriptive sheets upon which so much dependence seems to be placed. Indeed, it almost sounds as if *they* do the teaching rather than the teacher. Instead, they point the way. The work itself does the teaching; the teacher standing ready to pull the students out of quandaries that prove too much for them. The teacher must also check the conclusions the students are forming, to be sure the proper educational objectives are being reached. Otherwise dependence is placed upon the printed notes. Work sheets for those tasks repeated time after time by different students will describe completely each detail. Other sheets which prescribe one time tasks, such as the construction of a specific model, may be made in outline, omitting such key facts as the name of the

model to be made and the type of material to be used. By filling these blanks, differently for different students, various laboratory exercises can be made from the same printed sheet. A card catalog can be kept of names of available models to fill in the blanks and the pupils allowed to choose for themselves the one they wish to make. If the model, when completed, is acceptable for the general collection, the pupil would be asked to fill in his name on the card and the date the model was made. The card would then be transferred from the "Model-to-Make" file to the "Models-Already-Made" file.

Naturally all pupils must acquire a foundation for more advanced and more original work. The sheets on each type of exercise should be kept by the pupil in a notebook for a permanent record of what he has covered in the course. Many more fundamental and advanced exercises should be available than any one pupil could be expected to perform. This abundance allows the teacher to choose suitable material for the individual pupil, based upon his abilities and interest. It has also another important advantage. A teacher may require a certain number of exercises to be performed from a list containing two or three times that number. This forces the student to consider the importance, difficulty and attractiveness of the exercises he wishes to perform. By considering his choices the teacher has still another means of judging his abilities and ambitions.

Before we leave the mathematics laboratory, however, let us take a closer look at it.

It is a large room, used for nothing other than a mathematics laboratory. Here may be left, from day to day, the equipment and materials used in many experiments without fear that they will be destroyed or displaced by meddling. It must never be just a corner of a room used for some other purpose or sight will be lost of the fact that the experiments performed there are performed for their mathematical content, and not to help teachers of other courses, nor to give pupils the extra work in laboratory manipulation they have not been able to do elsewhere.

Specifically we see, along the windowed wall, a sink and table for plaster of paris work and a table for wood and metal work. Against the wall opposite the windows are two cases, one for exhibits, the other for books. One of the end walls supports shelves and cabinets for the storage of apparatus, the other, map stands, storage cabinets to hold posters and charts, and a file cabinet. In one corner is the desk of the teacher, facing two library tables, for student study and paper work, one physics table with gas and electrical fixtures, one work table, and two drafting tables. In addition there are two blackboards, one flat and one spherical, and a bulletin board. Here, as simply as that, we have our Mathematics laboratory.

V. Additional Material for Senior Year Mathematics

If the preceding discussion leaves the impression that all present day teaching of mathematics should be scrapped let us finish by mentioning some topics which can be used in a senior course of mathematics as taught in the ordinary manner. Most of these topics come from the experiences of the writer as he has watched others in the army struggle with the application of mathematics to every day work. These soldiers were learning mathematics because they needed it to do their job. They never would have learned the theoretical mathematics in school.

First, however, let us discuss a few natural questions about the course. What are the advantages of new subjects? Simply that mathematics growing from a series of practical problems is better received and more easily learned than mathematics presented as an exercise in logic or a series of brain twisting problems the answer to which the teacher knows—and the pupil must find. What is the connection between the philosophy of this

new senior year course and the idea of the mathematics laboratory course just discussed at length? Both demand the pupil do something with this mathematics *while* he learns it, and not merely pile up mathematical knowledge on the faith that someday all this magnificent knowledge will come into its own and he will bless the teachers who led him through the mysteries. After watching skilled mathematicians (and mathematics teachers) trying to use their mathematics in the army, I suspect the uses of mathematics are not taught because they are not known. Teachers would benefit greatly if they went out and *used* mathematics and discovered how others use it, rather than remaining content with their contemplation of its inner beauty, smug in its completeness and self-sufficiency.

Who would take this course? Since it takes considerable time to delve into applications, this course will not give as firm a foundation for future mathematics; therefore, it will be of greatest benefit to high school seniors who do not expect to continue with mathematics. It might also be adapted for others, and eventually be recognized by colleges as a course comparable to those now given.

What provisions for credit should be made in the curriculum? The course should be given for a whole year as two half courses. It should be parallel to the usual offerings of solid geometry, trigonometry, and advanced algebra. With the prerequisites of one year of plane geometry and two years of algebra this course could be allowed the same amount of credit toward graduation as any other course. Acceptance of it by colleges will have to wait for its full development.

A lengthy text book would be required to tell how these subjects should be taught or to mention what the complete contents of each course should be. But here we are interested only in the type of subject.

What subjects then could be used to form units in this new course? The list to be discussed is far from complete, it is only a set of suggestions to be tried out. Units could be organized, replaced, and shifted from year to year to give variety to the teacher's viewpoint. It is very important for each teacher to teach something new every year—and here is the opportunity.

Surveying and map making could be studied from the book and later applied to problems about the school. This is directly opposed to the approach of the laboratory course. Neither method is the better. For quick, complete, and *scholastic* results the "books first and applications later" method is better, but for many the laboratory approach is the only one which will give any results. Try both methods, there may be room in the school for each of them.

Celestial navigation, as used in ocean travel and aviation, is too difficult to teach in the laboratory. A theoretical approach with models, films, spherical blackboards and problems in location at night would work much better. Little imagination is required in order to realize the importance of these problems for later application.

The prediction of airplane flight or the "antiaircraft problem" is a complicated combination of solid geometry and trigonometry requiring a fairly long time to develop. Its final objective, however, could be kept in mind without much trouble.

Nomographs are becoming more and more important to engineers, and might be combined with industrial graphs to form a teaching unit. It is amazing how those in business are unable to show data on production, workload, personnel, fiscal requirements, unit cost information, or time studies in a readable form. They depend upon experts to make and interpret graphs for them when they could, without too much trouble, learn the processes themselves.

More and more groups are finding the analysis of technical statistics required in their work, but as yet no suitable course is available in either secondary school or college. Actually it is a subject well within the understanding of the high school student. Different kinds of averages, meas-

urements of dispersion, and some attempts at predicting future behavior from present data would be a good core of material to begin with.

If the group is interested in science, and equipped to wrestle with the material, a discussion of radio wave propagation would combine the algebra of formulas, the physics of weather prediction, nomographic charts, two or three dimensional graphs, no small amount of electricity, and some discussion of Geography and Geology. This is a topic not to be approached lightly, but it has much material and many rewards for close study.

On the other hand, a unit in consumer Mathematics could be enjoyed, used, and understood by all. The basic discussion could be about percentage and the various types of time-payment plans. Fallacies in reasoning would be found which conceal absurd rates of interest in simple sounding schemes. Other discussions could show the advantages of buying in large quantities, up to a certain point. Home budgets, insurance plans, annuities, and retirement policies could be analyzed and discussed.

Without straining to include many additional topics, these illustrate the types of units which could be organized and experimented with to show pupils that all mathematics textbooks are not translations from the Greek, nor all topics in the mathematics department the bitter wine of mathematicians whose greatest fear was controversy, and who, therefore, avoided all contact with the world about them. Teachers who find their refuge and security in the steadiness, and changelessness of mathematics through the ages, should be turned out-of-doors in the middle of the winter, with no shoes, and only a transit with which they must close a survey.

VI. Conclusion

As the reader has already discovered, the foregoing discussion stems from the ideas collected by a mathematician, who is also a mathematics teacher in civilian life. During his service in the army he has come in contact with many people who use mathematics in many different ways. Working with them, observing them, and listening to them has been a fortunate experience. Finally, it became necessary for him to put his ideas on paper.*

Well—here they are.

* Many thanks are due to Robert F. Purinton for his help in writing this article. His interest in the subject and willingness to discuss it have constantly forced ideas away from vagueness and toward clarity.

The Eighteenth Yearbook

The following review of the Eighteenth Yearbook of the National Council of Teachers of Mathematics on "Multi-Sensory Aids in the Teaching of Mathematics" appeared in the University of Michigan School of Education Bulletin for October 1945. We think it will be of interest to our readers. The yearbook can be obtained postpaid from the Bureau of Publications, Teachers College, 525 West 120 Street, New York 27, New York, for $2.00.—Editor.

In comparison with aids in some of the other subjects, there seems often to be a bleak paucity of illustrative materials in mathematics classrooms. This fine yearbook, prepared by a committee under the chairmanship of Professor E. H. C. Hildebrandt of Northwestern University, shows what can be used and how to use it. The many articles cover a wide range of topics and the book, appropriately, is well illustrated. There are extensive lists of sources. Principals and superintendents may well secure the book and encourage mathematics teachers to enrich the illustrative aspects of their teaching.

Universal Military Training in Peace Time*

FOREWORD (prepared by Professor M. H. Stone, Chairman of the War Policy Committee): The War Policy Committee of the American Mathematical Society and the Mathematical Association of America was formed to study the many questions of professional and scientific policy arising out of the war. No subject has been of greater interest or more vital concern to the Committee than the relations between scientific effectiveness on the one hand and the military requirements of the nation on the other. A most important aspect of this subject is treated in the report on Universal Military Service in Peace Time which is now made public. This report, prepared some time ago by a special subcommittee, is directed in the main at points upon which mathematicians as such are particularly qualified to express informed opinions. Whatever view may ultimately prevail concerning universal military training in peace time—and it should be emphasized that there are many citizens, mathematicians included, who doubt the wisdom of introducing such a peace-time military program—it is clearly of the first importance that no program deleterious to the scientific and technological vigor of the nation should be adopted. The report deals frankly and in detail with this vital segment of the problem now before Congress. In offering the recommendations of the report as a professional contribution to the current discussion, the War Policy Committee hopes to render a modest public service strictly within the natural sphere of its activity.

* A Report of a Subcommittee of the War Policy Committee of the American Mathematical Society and the Mathematical Association of America prepared in July 1945, by a subcommittee consisting of Professors W. L. Hart (Chairman), Saunders MacLane, and C. B. Morrey, Jr., and approved by the War Policy Committee, the Council of the American Mathematical Society and the Board of Governors of the Mathematical Association of America.

1. INTRODUCTION

In the Congress of the United States, bills are under consideration (S. 188 and H.R. 515, companion bills) which would prescribe a year of military service in peace time for all young men of suitable physical qualifications. These bills present the nation with a proposal for a fundamental new departure in our national life and it is appropriate that the widest discussion should occur concerning all sides of the situation which would result. It is natural that the greater part of the arguments pro or con about S. 188 in the public press should deal with military, internal political, and international aspects of the matter. Various spokesmen of the general field of education, particularly at college level, as well as of specific fields of learning have likewise discussed the measure under consideration. It is the main purpose of this report to state opinions concerning the impact of compulsory military service in peace time on those phases of our national life which are of primary significance to mathematicians and which lie in areas where we may speak with special authority. In carrying out this aim, it is natural that at times we shall phrase remarks from a general educational viewpoint not primarily associated with the field of mathematics. A summary of the opinions advanced in the report is to be found in Section 11.

2. ORIENTATION OF THE REPORT

In the discussion of any measure like S. 188, we may conceive of a division of interest between two major questions. First, one may ask, "Should the United States have universal military service in peace time?" Second, "If Congress is to pass such an act, what provisions should it contain in order that the greatest possible good should be obtained for the nation as a whole and for the young men who will perform the service?" We realize that these questions are considerably in-

terrelated but, for our purposes, we shall act as if they are relatively independent. It is probable that the decision on the first question should and will be reached mainly on the basis of significant national and international considerations of a social economic, political, and military nature. In this connection it is likely that only small weight will be given to those points where our opinions as mathematicians or educators would be of any special importance as compared to those ideas we may express individually as mere citizens. Hence we have decided that, in regard to the first question and the most intimately connected phases of the second question, we shall make only a single statement.

> We believe that a decision about introducing military service in peace time should be reached only after an investigation by a carefully selected commission, preferably appointed by the President of the United States. We recommend that this commission should include not only representatives of the armed forces but also individuals from all other important sections of our population, with a strong representation from the fields of industry, science, and technology which have been of such great importance during the present war. This commission should have the wide objective of considering what combination of universal military service, scientific development work on weapons of war, and associated industrial and educational measures would be best adapted to insure safety of the nation.

The balance of this report will be concerned with matters particularly germane to a discussion of the second of the two major questions which we have mentioned, although we have no intention of considering all its aspects. Our remarks will be phrased under the explicit assumption that *universal military service in peace time will be adopted*. However, this view point is not to be taken as evidence that we desire or expect a bill such as S.188 to become law.

For future reference, we note that the present bill S. 188 includes the following features.

2.1. The bill omits specification of all subsidiary values as justification for military training and is based only on reasons relating to national security and the prevention of dissipation of training resources and experience.
2.2 The bill contains provision for a period of four years (ages 18 to 22) in which the individual may select the year for military training. A high school graduate with his parents' consent may volunteer to start his year of training while he is still 17 years of age.
2.3. The year of training will be used by the Army and Navy as a basis for selecting non-commissioned and commissioned officers.

3. Background Relating to the Present War

The importance of technical developments of new weapons and tactics during the military operations of the last few years has emphatically shown that science has made a fundamental contribution to the military power of the nation. More particularly, the abstract science of mathematics has been put to effective application in a surprising number of fields, including even the extensive use of mathematicians in operational analysis work overseas. These facts are important in the formulation of an intelligent long term policy for maintaining the nation's military strength. It can safely be said that the military potential of a country does not depend solely upon the size and character of its armed forces. This potential also depends largely upon the technical and industrial talent available and upon the vigor, ability, and training of its scientific and technical experts.

> Long range military planning, therefore, requires that able young men be encouraged both in preparing themselves in basic scientific and mathematical knowledge and in acquainting themselves with the ways in which this knowledge can be put to use in time of war or national emergency.

With the preceding viewpoints in mind, we deeply regret to note that the man-

power policies of the United States during the war have not been well formulated with respect to the utilization of men trained in science and technology or potentially able to assimilate such training. Draft deferments for the purpose of obtaining certain varieties of technical training have been at times difficult and often impossible to obtain. Some valuable technical men have been wasted in non-technical activities and others would have been wasted had it not been for the energetic efforts of representatives of science and technology in prodding reluctant or unenthusiastic government bureaus into action. During the war the United States has followed a sadly shortsighted and unintelligent policy with regard to science and technology by not providing for appropriate annual increments of undergraduate men to be trained in those fields. There are facts at hand which indicate that England and probably Russia have not made a similar error during the war. We agree strongly with the many opinions which have recently been expressed concerning the necessity for prompt resumption of training for the scientific professions, even before the end of the war with Japan. We believe that any additional delay in provisions for such training may have fatal effects not only on national health and the technological side of our economy but, also, on our means for future national defense. These remarks are well justified by the established facts about the present and potential shortages of physicians, dentists, Ph.D.'s in the various fields of the physical sciences, and technologists. The preceding background leads us to the following conclusion.

> We consider it very important that universal military service in peace time should not be planned in such a way as to interfere with efforts, first, to eliminate as quickly as possible the present shortage of scientists and technologists and, second, to provide for a continuous generous supply of such essential categories of trained citizens in the future.

4. An Attitude About Exemptions from Military Service

It has been the traditional American viewpoint that all men should be treated as nearly alike as possible in fundamental respects whenever a call for universal military service is issued by our Government. It could be argued that this was a major cause for some of the shortsighted man-power policies of the present war period which have been referred to previously. However, we believe that it would have been possible for Selective Service to have been administered during the war without violating the specified tradition and, also, without introducing various major evils. This democratic viewpoint is so deeply seated that, regardless of whatever arguments might be presented against it, neither the general public nor the affected young men could be expected to give friendly support to measures for creating numerous exempt classes of the population in case universal military service in peace time should be adopted. Realistically, we appreciate that any request for the exemption of mathematicians or physical scientists, for instance, could be expected to induce requests for the exemption of social scientists, and scholars in almost all other fields of learning, as well as representatives designated by innumerable pressure groups outside the field of education. The result of such requests for exemptions would undoubtedly be the creation of an ironclad policy of *no* exemptions. The requests might even develop an unsympathetic attitude among the directors of military service with respect to rational plans for differentiation of training, which we propose to emphasize in place of exemptions.

> We shall proceed, then, with the explicit premise that it would be undesirable to suggest the exemption of men with special mathematical talent and that each man, regardless of his abilities, will have to spend one year of his life in military service.

5. A Liberal Interpretation of the Words "Military Service"

The present rate of increase in the military uses of science and technology convinces one that success in any future war will be dependent on continued progress in science and on the existence of a large reservoir of trained technologists. Thus it is essential that any program for universal military service should be consistent with associated plans for the development of our scientific and technological potential. It follows that the final program should not be organized with the narrow objectives of merely disciplining masses of men and preparing them for handling existing weapons of war. Hence, the term "military service" should be interpreted in a liberal fashion.

> "Military service" should include not only the usual routine service in the Armed Forces but, also, various other highly technical forms of training which are just as essential for enhancing the military strength of the nation. These types of service should be made available for properly qualified young men under some studied system for differentiation of training in accordance with ability. In particular, the system should provide appropriate advanced types of service in the case of the small but important group having special aptitude for and training in mathematics, the physical sciences, or technology.

6. Candidates for the Most Highly Technical Military Service

During the present war, under Selective Service the Armed Forces have made intelligent efforts to place the inductees into various enlisted classifications in accordance with the abilities shown by the men in civilian life and in various tests given by the Armed Forces. Also, similar efforts have been made to locate men with appropriate backgrounds as candidates for commissioned ranks in various branches of the Armed Forces. It should be anticipated that these commendable practices in the direction of differentiated training would continue under universal military service in peace time. The Committee then directs the main force of its recommendation for differentiated training at the cases of the exceptionally superior young men who, let us say, have the requisite mathematical and scientific aptitude and inclination to become Ph.D.'s in mathematics or a physical science, or to advance to the highest levels of attainment in an engineering field. Under the reasonable hypothesis that less than one-third of the men in our population with such potentialities finally pursue careers in the indicated fields, we are led to estimate the number of men of this sort available in any annual age group at about 3000 men. If we should attempt to sift them out of the enormous annual group of high school graduates by any existing devices for the prediction of mathematical or scientific aptitude, the percentage of error in diagnosis would be very high. It is likely that we would have to earmark about 20,000 boys annually in order to be reasonably certain that the final group would include 90% of the 3,000 boys whom we have mentioned. Such an attempt at selection of 20,000 boys would entail severe difficulties in the field of educational measurements and also many later complications if the members of the earmarked group were to be given individual attention during the next four to seven years. Our final plan will indicate a procedure for obtaining such candidates which appears more simple than an attempt to earmark them at an early age. At present we offer merely the following conclusion. Among the boys who graduate from high school in any year there is a group of about 20,000 with such high mathematical and scientific aptitude that they deserve special consideration under any plan for universal military service. This group will hereafter be referred to as the *select technical group.*

7. Advantages of Deferment of Military Service for the Select Group

It is likely that any boy in the group would graduate from high school by the

time he is 18. Regardless of considerations related to military service, the rarity of the aptitudes present in the group makes it desirable that each member should be shielded from unnatural influences which might lead him away from the fields of science and technology where his abilities would be so valuable to the nation. We believe that he should be encouraged to carry out any latent inclination to enter upon scientific or technical training. Immediately after graduation from high school, an interruption of a full year in the educational process might eliminate or dim his embryonic urge to scientific study. The interruption would also cause an undesirable break in the important preliminary stages of progress in mathematics and science. Moreover, a year of military service at age 17 or 18 might expose the boy to certain military, social, or industrial activities which, although appealing to a youth, could lead him after a few years to an undesirably low terminal point outside of advanced science or technology, with a corresponding loss to the nation.

If our attention is now fastened primarily on differentiated training at a high level under military service, we can likewise argue that a delay beyond age 18 in this service may be desirable for a boy in the select group. When he graduates from high school, he will not yet possess the requisite mathematical and scientific background to be eligible for or to obtain maximum good from many of the high grade types of differentiated service which can be made available. Before he receives college training, his possibilities might be merely those of an exceptionally intelligent young man with latent scientific abilities and interests but no particularly useful background knowledge. He might even gain a permanently wrong orientation with respect to various aspects of military service where he should eventually become most useful to the nation in case of another war.

The preceding remarks lead us to advocate the following flexible provision, similar to Item 2.2, in any final regulations governing universal military service in peace time.

> The directors of the program should have the power to approve a request for deferment of the year of service for any young man when this is deemed best for the interest of the nation. In particular, continued efficient progress with high achievement in the study of mathematics, a physical science, or technology, even through the stage of a Ph.D. degree, should be considered sufficient cause for such deferment.

Conceivable complications in the administration of the rules for deferment should not be considered a deterrent to the adoption of the preceding recommendation. It should be possible to develop a smooth routine for such a process in peace time which would be superior to the method employed with respect to draft deferments of scientists and scientific students during the war. In this connection it is pertinent to observe that a proposal for special treatment immediately under Selective Service for a select group of 20,000 was presented to Congress recently.

8. Suggestions Concerning Highly Differentiated Service

We proceed under the assumption that the deferment recommendation of Section 7 could be adopted. Then, we propose the following rough outlines of a plan for implementing the suggestions about differentiated service.

> 8.1. A board should be created to canvass the possibilities for in service training on intermediate and higher technical levels for properly qualified applicants and to supervise plans for instituting such training. This board should involve civilian representatives of the fields of mathematics, the physical sciences, and engineering, together with representatives of the armed forces.
>
> 8.2. The search for training locations should cover the various laboratories, research divisions, arsenals, and other technical activities of the Army and Navy. Also, the search should extend into associated private industries and related curricula and research in universities. As a mere sample of possibilities we mention the following types of

training which might be given to properly selected groups each year.

8.21. An advanced course in meteorology for the future use of the Army or Navy, along the lines of the curriculum presented at five major universities in the United States during the early years of the war, and associated field experience.

8.22. In service training at the Ballistic Research Laboratory of the Army Ordnance Department at the Aberdeen Proving Ground.

8.23. In service training in the artillery fire control division at an ordnance plant.

8.24. In service training at a laboratory of the Signal Corps, or at an aeronautical research laboratory of either the Army, the Navy, private industry, or a university.

8.25. Study of the mathematical and operational phases of cryptanalysis in the intelligence division of the Army or Navy.

8.3. For each of the training possibilities, the Board should decide upon appropriate academic prerequisites to be demanded of any man who is to qualify for the opportunity. These prerequisites should probably be much more extensive than for any similar training during the war, because in peace time interested men would have ample opportunity to obtain the necessary background. A pamphlet should then be published listing the available varieties of technical training which might be taken as "military service" so that interested boys, even early in their high school life, could begin to orient their school work with respect to the future opportunities in military service. The pamphlet should urge boys of proper aptitudes to defer their entrance to military service until they gain the prerequisites for the type or types of service which appeal to them.

8.4. If administratively possible, assignments to any one of the most exacting and desirable varieties of training should be made on the basis of applications asserting definite interest in the field involved, as well as possession of the necessary prerequisites.

8.5. In addition to the technical training experience mentioned previously, an appropriate amount of routine "boot" training should be given either simultaneously with the technical work or in a separate period of time during the year of service.

8.6. The directing Board should examine the possibility of granting commissions in the Reserve Corps of the Army or Navy for satisfactory completion of certain types of technical training which would be available to the most highly qualified young men during their year of service. Acceptance by a young man of the opportunity of such training should not commit him to accepting such later commission as might be offered in this connection.

9. Civilian Educational By-Products of Universal Military Service

If the interesting nature of the various technical types of differentiated military service, at low or intermediate as well as at high levels, and the corresponding academic prerequisites are properly advertised, strong repercussions might occur in the field of secondary education. It is probable that boys of ability and their parents would then demand maximum mathematical and scientific opportunities in the secondary curriculum for students of proper ability. Also, regardless of the possibility of differentiated training, it is our belief that a requirement of a year of military service, with the attendant delay in the study of the professions, would place an added demand for educational efficiency on both the high schools and colleges. The knowledge resulting from a brief exposure to substantial secondary mathematics or science is easily forgotten. On the other hand, a long exposure to these fields gives the student a body of content which, even though it should remain dormant during a year of military service, would be effective in the next year after a brief review.

Hence, if universal military service is adopted, added efforts should be made in the secondary field to expose the better students to as much substantial mathematics and physical science as possible.

10. Educational Responsibilities of the Armed Forces Resulting from Universal Military Service

Even without the delay which would be caused by military service, the time for

completing professional training encroaches on the age of greatest productivity and ingenuity as well as on the normal age for assumption of family responsibilities. Hence, under universal military service, the Armed Forces will be presented with a challenge to compensate as much as possible for the year of delay which would be introduced. It is obvious that the handicaps suffered by a young man due to an interruption of a year in his formal education would be considerably lessened if he should systematically carry on correspondence study during the year.

If universal military service is adopted, the Armed Forces should offer to the men in training the fullest cooperation and encouragement in the extension of their formal education by means of correspondence courses. At the college level, we recommend that this be done by paying the registration fees for men who take and diligently pursue courses offered by regular college correspondence departments, rather than through such an agency as the present Armed Forces Institute.

11. Summary

In the development of this report we have emphasized the following points and recommendations.

11.1. A joint civilian and military commission should be appointed by the President of the United States to study the problem of universal military service in peace time before final action is taken on the matter.

11.2. An expression of opinions by the American Mathematical Society and the Mathematical Association of America should be focused on constructive suggestions about the administration of universal military service if it should be adopted and about the solution of resulting educational problems.

11.3. No outright exemptions from universal military service should be requested.

11.4. The required military service should be highly differentiated in accordance with the aptitudes and training of the young men involved, with emphasis on exceptional differentiation for those few with the greatest technical abilities.

11.5. A system for deferring the year of military service should be instituted so that gifted young men might prepare themselves for advanced varieties of differentiated service before entering the Armed Forces.

11.6. The possibilities for technical varieties of differentiated service should be canvassed by a joint civilian and military board. The resulting training programs with the corresponding academic prerequisites should be well advertised among high school boys, their parents, and the teachers and administrators in the secondary field.

11.7. If universal military service is adopted, the field of secondary education will have the added responsibility of increasing the efficiency and quantity of instruction given in mathematics and physical science to the students of better than average ability.

11.8. The Armed Forces should encourage young men to continue their education during military service by taking correspondence work through regular school channels, and the Armed Forces should pay the costs of such study if a man carries it through diligently.

Geometric Spelling

In my youth someone once told me a very interesting way to spell TOBACCO. When I came to the tobacco section of Maryland to teach, I remembered it, revised it slightly and passed it on to my Mathematics classes. It is this:

**One perpendicular and a circle complete, two semi-circles with a straight line to meet, an acute angle triangle with line segments as feet, two semi-circles and a circle complete.—
By Mary V. Haile**

Mathematics Interest—Fundamental or Not?

By Paul Torrance
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Several years of teaching mathematics and counseling high school and junior college students has often influenced me to wonder if there is such a thing as a "fundamental" interest in mathematics—or any other subject for that matter, but especially mathematics. Has a student by the time he has reached high school or junior college even acquired such a set of likes and dislikes that he has become the kind of person who cannot become interested in mathematics on the one hand or is the kind who is naturally intensely interested in it on the other hand?

In vocational counseling it has been fairly well established that we cannot rely too much upon the expressed interests of our counselees. These expressed vocational choices or likes and dislikes we have found are often very superficial and may be highly colored by a single incident which has occurred in some connection with that interest. It is of course recognized that "an interest is not a separate psychological entity but merely one of several aspects of behavior" and is an expression of satisfaction but not necessarily one of efficiency. Research has indicated, however, that there is some relationship between interest and ability and certainly between interest and vocational success.

Could the same be true regarding interest in school subjects? Does the student who says, "I hate mathematics!" really have this strong a dislike for mathematics or could he be fundamentally the kind of person who would really like mathematics if given the proper stimulation and "reconditioning"? It seems possible that such a superficial dislike could have been contacted through some such factor as parental attitudes, a poor or unenthusiastic teacher somewhere along the line from kindergarten up, an uninspired or unenthusiastic teacher at the present time, or some single unfortunate experience in relation to mathematics.

On the other hand, what of the student who says, "I am crazy about mathematics!" Does he really mean it or might not this too be only "skin deep"? Is he really the kind of person whose interest is genuine and who will continue to like mathematics? Or is his interest only a superficial thing stimulated by superficial rewards, social or family approval, or some other external factor or factors?

The research concerning vocational interests has been quite extensive and there are now on the market several quite useful vocational interest tests or inventories. Over ten years of extensive research in which many thousands of men and women have participated are back of Strong's *Vocational Interest Blank* and it has become a very useful tool of the vocational counselor.

The research regarding interests in school subjects does not seem to have stimulated very much research. Most notable have been those who have employed an attitudinal approach to interest in school subjects. In such an approach, again there seems to be too much opportunity for the superficial aspects to enter the picture—such things as a highly colored attitude stimulated by teacher-pupil conflicts, overemphasis, parental prodding and the like.

The present investigation undertakes to employ a somewhat similar approach to interest in mathematics as Strong has employed to vocational interests. Strong contends that knowledge of several hundred likes and dislikes provides a much better basis for estimating past and future behavior when based on the statistical

analysis of hundreds of cases. The inventory used in this study is composed of one hundred items selected largely from the most significant items on the scale for mathematicians from Strong's *Vocational Interest Blank*. The subjects of the study were 500 high school and junior college students enrolled at Georgia Military College in 1944. The form in which the inventory was administered is almost identical with that of the Strong Inventory. In compiling a tentative scoring key, a score of two points was assigned to those items weighted most heavily on Strong's scale for "Mathematician" and one point to items considered significant but not so, heavily weighted. Scoring was made very simple by using two stencils, one for the items having double value and another for the other items.

After the inventory had been administered to about 500 students and scored experimentally a survey was made of the mathematics records of those ranking in the upper ten per cent on this scale and those ranking in the lowest ten per cent. Of those ranking in the upper ten per cent, only three had ever failed any mathematics even for a quarter and only one had failed a course. The majority of this group had averaged "A's" in all high school mathematics and the average of the whole group was "B." It was also interesting to note that the two students making the two highest scores were clearly the two most outstanding mathematics students in the school; both took the College Entrance Board Examinations and the first ranked in the upper one-half per cent and the other ranked in the upper seven per cent in this competition.

Of those in the lowest ten per cent, only four have escaped thus far without failures in mathematics; however it was noted that each of these four students rated considerably above average in aptitude and that their grades without exception were much lower in mathematics than in other subjects. The fifty students in this lowest ten per cent have already failed mathematics for a total of 165 quarters or an average of 3.3 quarters per student and thirty-five per cent of these were only in first year high school mathematics.

The scale was also administered to the members of the faculty and consistently higher scores were made by those who had majored in mathematics or had studied considerable mathematics in college.

As a further check on the validity of the measure a study was made of 88 students who had just taken the *Kuder Preference Record*. When the scores on this scale were correlated with the scores on the "computational" interest scale a coefficient of correlation of 0.31 was obtained and when correlated with the "scientific interest" score a coefficient of correlation of 0.50 was fund.

The scale was also employed in a special study of the prediction of success in plane geometry. The coefficient of correlation between the score on this scale and success in plane geometry as measured by teachers' marks was 0.40 and as measured by the *Cooperative Plane Geometry Test* was 0.30. When it was combined with other prognostic measures it improved their predictivity considerably in most cases. In this study, although the scale did not correlate very closely with success, it seemed to have significance for counseling especially at both extremes and seems to correlate more closely with grades assigned by teachers than with achievement tests. It was also indicated that it seemed to be helpful in diagnosing difficulties of students lacking in this interest and also in identifying those whose interest in mathematics has been formant. If predictive measures are to be used clinically in prediction, remediation and counseling it seems to be one of the most important factors. Wherever there are wide discrepancies between aptitude and achievement in many cases the interest factor seemed to be an important one—the measure which seemed to explain the discrepancy.

After the items in the inventory had been scored experimentally the validity

of the individual items for measuring interest in mathematics for high school and junior college boys was checked by comparing the responses made by the fifty students having the highest scores with the fifty having the lowest scores. By a process involving this procedure the scale has been reduced from 100 to 75 items which give evidence of being valid for discriminating between mathematicians and non-mathematicians as determined by Strong's researches and between teen-age boys who have strong interest in mathematics from those who seem to have weak interest. The items eliminated, while they distinguish between the interests of men who are mathematicians and those who are not, do not have this ability when applied to teen-age boys. Maturity and experience are probably the determining factors here.

Examples of how these differences operate may be noted by examining the twenty-five items eliminated:

Aviator
Army Officer
Certified Public Accountant
Reporter, sports
School Teacher
Secretary, Chamber of Commerce
Secret Service Man
Statistician
Military Drill
Music
Hunting
Educational Movies
Golf
Art Museums
Entertaining others
Teaching Children
Teaching Adults
Meeting and directing people
Drilling soldiers
Methodical work
Energetic people
Optimists
Carelessly dressed people
Athletic men
People who have done you favors

Examination of these items reveals that in many of them typically "adolescent boy interests" are operating—an expressed like for the exciting and dislike for occupations and activities accounted dull in the estimation of adolescent boys. For example more boys who had high scores thought that they would like to be aviators than those who have an aversion to mathematics even though the reverse is true when applied to mathematicians and non-mathematicians. The same was true for army officer, military drill and drilling soldiers. In these cases the environment of a military school may have been a determining factor. In other cases lack of vocational information may have been an important factor as possibly for statistician, certified public accountant, secretary of Chamber of Commerce.

Although the implications are not clear the evidence seems to indicate valuable possibilities for the development of interest scales for various subject matter fields along lines different from the attitudinal approach of Remmers and others. Such a tool should supply information of tremendous value in clinical counseling and in the selection of curricula for individual students. In addition such a project should supply data for the construction of a tool for the vocational counseling of high school boys; it seems fairly certain that the Strong *Vocational Interest Blank* does not serve this purpose too well; some of the reasons might be those indicated in this study.

Be sure to attend the Annual Meeting of The National Council of Teachers of Mathematics at The Hotel Statler in Cleveland, Ohio on Friday and Saturday, February 22 and 23!

The Fallacy of Social Arithmetic

By Harry G. Wheat
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It is axiomatic that when a theory concerning education sounds too good the teacher should be suspicious of it. For a decade the theory of *social arithmetic* has been sounding too good,—much too good. It embodies an easily understood method of instructing pupils in arithmetic. It suggests an already familiar content. It supplies meaning much as a gift to be accepted instead of an achievement to be striven for. In short, *social arithmetic* is a ready-made answer to the problems that confront the teacher. He does not need to turn to the back of the book. Indeed, he is supplied the answers before he meets the problems. It is about time for the teacher to become suspicious. It is about time for him to inquire what *social arithmetic* is and how it got that way.

1. *What is social arithmetic? Social arithmetic* is the arithmetic that grows out of the social situations with which the pupil comes in contact. In each, there is a number element which is a characteristic feature. In each, there are number concepts which play important parts in determining the total form of the situation. These—the number element and the number concepts—stand out so clearly as to make them easy to recognize. (That is to say, the mature teacher can recognize them with ease; therefore, so can the immature pupil.) All that is required is that the pupil have experience with social situations. The logic is simple: (1) Number is present in social situations; (2) experience with the situations implies experience with the number elements therein; (3) number concepts (like wet paint) adhere upon contact.

Moreover, *social arithmetic* is the arithmetic of meaning. Here, again, the logic is simple: (1) Experience with social situations provides meaning; (2) the meaning so acquired attaches to whatever is derived from contact with the social situations; (3) number concepts are so derived; and so (4) the experiencing learner acquires both the number concepts and their social meanings.

The theory of *social arithmetic* greatly simplifies the procedures of instruction. Since the pupil's experiences have already provided number concepts and their meanings, all that is required of the teacher is to make the pupil aware of what he already possesses. The reason for the requirement is not entirely clear to the true believer, yet the requirement is generally accepted and met. It is accepted because certain computational skills are considered important and awareness of their value in advance is said to provide motivation. It is met through the process of recalling experiences already gained.

There are, of course, gaps in the experiences of the pupil. These the teacher must aid the pupil to fill, if not always with direct experiences, then with experiences that are indirect. Thereby the pupil may round out his number concepts and acquire the motive for the needed skills which otherwise might be by-passed.

How may the teacher know that gaps exist in the experiences of the pupil? How does he know what skills are needed? By what procedure does he undertake to develop skills? When we raise these questions, we are at the point of discovering why *social arithmetic* is only one of the two branches of the school arithmetic that is advocated by the social arithmeticians. We discover beside *social arithmetic* a divergent branch which is called, of all things, *mathematical arithmetic*!

2. *What is mathematical arithmetic?* It appears at first glance that the use of the term *mathematical arithmetic* is just an ex-

ample of "gilding the lily." Or it seems that the term has no more use, or value, or sense than would "linguistic speech" or "harmonic music." But when we look closely, we discover that *mathematical arithmetic* is called *mathematical* presumably because it is *non-mathematical.* The logic here is somewhat obscure, but the fact is plain. Though *mathematical arithmetic* is mathematical in form and sequence it is generally non-mathematical in substance, rather in its lack of substance.

The substance, that is, the meaning, of arithmetic is not looked for in *mathematical arithmetic.* Why should it be, when it is held that it is the social phase that provides meaning? Do not number concepts issue from the experiences of the pupil with social situations? There is then no need to turn to *mathematical arithmetic* for ideas that are supplied elsewhere and by different methods. Thus we reach the paradox that the simplest of the sciences may lay no claim to meaning except such as issues upon it from the complexities of accidental experience.

But *mathematical arithmetic* has its uses. Without it the school course in arithmetic would not be complete. In the first place, it is a compendium of all the skills the pupil needs to acquire and it suggests the gaps in his everyday experiences which need to be filled by experiences that may be less direct. Secondly, it has a distinguishing method quite distinct from that of its social partner. The method of *mathematical arithmetic* is the method of drill,—meaningful drill, to be sure, with its meanings supplied from sources apart from it and outside of it.

Thus in our school arithmetic presented to us today,—in prefaces to textbooks, in courses of study, in discussions before and among teachers, and elsewhere,—as a bifurcated design. Thus is our school arithmetic a set of parallel paths over which the pupil must travel. The chief difficulty of the teacher appears to be that of seeing that the pupil is constantly stepping back and forth from one path to the other. There is, on the one hand, *mathematical arithmetic* with its method of drill, and, on the other, *social arithmetic* with its method of recalling and providing experiences. How to teach arithmetic is no longer a problem. It is merely a difficulty of maintaining a balance. Both phases of arithmetic must be properly emphasized, we are told; both paths must be traversed. Over the one, the pupil acquires skills without meanings; over the other, he gathers meanings without skills.

Not all who bill the school course in arithmetic as a double feature repudiate the mathematical half as a source of meaning. There are those who announce the mathematical half as the supplier of meaning and the social half as the supplier of significance. Thereby they lead themselves into error. Having given the social importance of arithmetic a prominence coordinate with meaning, they frequently find it reasonable and themselves impelled to consider the pupil's possession of the former outcome the motivating drive for his pursuit of the latter. Meaning thus becomes a consequence of significance. The step to subordination is short. Moreover, having adopted the social half of their program in order to supply significance, they thereby and at the same time take a position from which it is most difficult to envisage social situations as the developers of meaning elsewhere gained. The view they miss will be indicated in a later topic.

A curious process of reasoning justifies the arithmeticians just described in the position they take. At the outset, they repudiate *social arithmetic* as the product of an extremely narrow view. They will have none of it. Instead, they will have an arithmetic which is designed to develop meaning,—number meaning. At this point, they seem to regard their view likewise limited. It embodies thus far *only* the *mathematical aim,* or the *mathematical phase,* of arithmetic. For a complete arithmetic there must be added a co-ordinate *social aim,* or *social phase.* Finally, they

reach the point where they view their *social aim*, or *social phase*, in a superior light. Here, finally, they are ready to justify their *mathematical aim* by the importance of their *social aim;* for, as they seem to size up the matter, it is the value of the *social phase* that makes it worthwhile to have the *mathematical phase* at all. It would appear to the outsider looking in that they are back with the original *social arithmeticians*,—not as far back, since they find a meaning in their *mathematical phase* which the *social arithmeticians* do not sense, but *back*, since the meaning they do find seems to them to be meaning of limited value.

3. *Why do we have social arithmetic?* For a generation, at least, the theory of *social arithmetic* has been with us. At the outset, it served as a means of determining the kinds and amounts of arithmetic to teach. Thus, the arithmetic that was considered "socially useful" was recommended to be taught. As a consequence, much of arithmetic was excluded, or stood in danger of being excluded, from the course. A contending theory held that certain relations existed between the parts of arithmetic, and that in an emasculated program these relations would be lost with the result that even the "socially useful" parts could not be effectively taught.

In time, the theory became much as it is today and not without reason. In the widely used drill method of instruction, meaning for the pupil was conspicuous by its absence. It seemed important to supply meaning. In *social arithmetic* number meaning was clearly evident—evident, as already suggested, to mature individuals; in this case, arithmeticians who were concerning themselves about the school course. Thus, since social situations held number meaning which they could apprehend, social situations were considered to hold number meaning which the untrained pupil could apprehend. Hence, the theory developed as one which should supplant, or supplement, the drill theory.

Today, we think we have outgrown our earlier shortcomings. We think we have a *new* arithmetic—a *meaning* arithmetic. This new arithmetic of meaning is intended to supplant the older styles with their somewhat violent oscillations between the extremes of the drill method and the incidental method. We will abandon these two older fashions, say the *Tenth Yearbook* and the *Sixteenth Yearbook*. We will have a *meaning* arithmetic—an arithmetic which is half *mathematical* and half *social*. Should not the teacher at this point be permitted an honest doubt? How do you get a new garment by sewing together two outworn pieces of cloth? How do you abandon two insufficient methods by combining them? A double negative may be an affirmative, but when you add two negatives the result still is negative.

4. *Can the doubter be interested in meaning?* The teacher who gives expression to such doubts as have just been indicated throws himself open to misunderstanding of his motives. He will be criticized as one who attacks the effort of the school to provide meaning and understanding for the pupil. Does he not attack *social arithmetic* which is the source of meaning? Does he not advocate *mathematical arithmetic*, which is nothing but skill and drill, and devoid of meaning except such as is brought to it? In answer, it may be submitted that his expression of doubt is the best possible evidence of his interest in providing meaning and understanding for the pupil.

In *mathematical arithmetic*, narrowed down as it now is in theory and practice, he discovers the same absence of number meaning for the pupil as do the social arithmeticians. In *social arithmetic*, interpreted as a way of learning and teaching, he discovers a like absence of meaning. In *social arithmetic*, interpreted as the experiencing of social situations, he discovers number meanings, but only such number meanings as the trained individual has first brought to them and then perhaps has developed in them.

The teacher who ventures a doubt about

the present bifurcated program in arithmetic that is gaining such wide-spread advocacy is interested in helping the pupil to gain number meaning. He sees a fallacy in the current theory of *social arithmetic.* He suggests that for the pupil to gain number meaning the pupil should be taken to the place where number meaning is to be found and there set to work.

5. *What is the fallacy?* The fallacy of the theory of *social arithmetic* has already been mentioned. It is the fallacy of reading into the attitude of the untrained pupil the point of view of maturity. In social situations number relations and their significant social values are recognized by the individual who has been trained to recognize them. To him social situations present number meanings and number significations which, if he wishes, he may abstract therefrom. There is thus some excuse for his belief that what he can recognize anyone can recognize, that what he can withdraw for study anyone can withdraw for study. It is a human frailty to assume that the world without appears the same to all. Notwithstanding, the assumption is a mistaken one.

Of course, there is number meaning in social situations for a person,—just as much as he has put into them. Also, just this much he can withdraw. It is something similar to withdrawing money from the bank. Anyone can do it, provided that first he has deposited money in the bank. The bank is not a source of funds, at least not for the common man. He must gain his wealth elsewhere. Then, and only then, may he make a deposit which, let us remember, is the necessary prelude to withdrawal. What one may withdraw is no greater than his deposit plus, of course, its accumulated interest. Similarly, there may be a growth of deposits of number meaning in social situations. But the first requirement is number meaning to deposit followed by the deposit of number meaning.

The analogy which has just been drawn does not suggest that because the local bank is not an original source of wealth a person should be opposed to banks. Banks are depositories of wealth. So are social situations. They, much more than do the banks, develop wealth. But, again like banks, they do not supply the original capital. Those who are wise seek to create their capital outside the banks through the employment of methods that are less simple than that of writing a check.

6. *How may the pupil gain number meaning?* The pupil gains the meanings of the number processes through guided, systematic study of number processes. The guidance and the systematization must be supplied by his teacher. The studying he must do himself. There is no shorter, simpler, easier, more glamorous road. The scenery along the way may have its attractions, but the pupil does not move ahead while he stops to enjoy the view.

Thus, to gain the meaning of division, for example, the pupil must study division. Let us observe how, guided by the thoughtful teacher, the study may get under way, and a point or two to which it may lead.

From the outset, what the division question asks must be perfectly clear. "How many twos are there in six?," for example, would need to be illustrated again and again by an actual division of a group of six into twos, and the pupil's attention would need to be guided to the required dividing that is demonstrated. Though the work of dividing may need to be done for him, the work of studying the division is nevertheless his own. Thus, from the very beginning, the pupil must determine the answers to all questions: "How many are here altogether?" "How many are here by themselves?" "And here?" "And here?" "Now, how many twos are there? Count the twos you see."

As the division question and what is required to determine its answer become clear, the pupil takes over the actual division of the group of six into twos—step by step, determining each partial answer until he arrives at the final answer. Somewhat

later, he can be sure of the final answer by short-circuiting some of his original work, that is, by *thinking* the division without actually doing it. Still later, he can be sure of the final answer entirely as a matter of thought, that is, without any reference at all to the actual grouping of objects. Finally, the pupil is able to carry his division,—the requirement of it, how to do it, and what it is when done,—to the social situation of six lollipops tied together and draped on the Christmas tree in groups of two.

The procedure in studying division is *not* from the *concrete* to the *abstract*. At the outset, the effort is made to reduce the concrete necessities to the barest minimum. Thus there is avoided all reference to the social obligations of sharing the lollipops, or the expenses of a picnic, or the profits and losses of an enterprise. The purpose is to exclude as much as possible all interesting distractions in order that the pupil may devote his attention to division. The objects used to demonstrate the division to be performed and how it is performed and to make possible the pupil's performance and study are, of course, concrete. But their concrete qualities are not referred to,—their color, shape, size, weight. Indeed, such objects as possess the fewest possible concrete qualities to attract are selected. Moreover, the teacher asks no questions about the objects. The pupil does not study them. The teacher inquires about division, and it is division that the pupil studies. The idea of division is *abstracted*, that is, "drawn from," its concrete surroundings and made the object of attention and study. Thus, at the outset, the pupil does not deal with the concrete. He deals with the abstract, namely, division and what he has to do when he divides. His performance is one of abstraction—abstraction of the simplest possible sort.

The study proceeds from simple abstractions to those that are less simple and more and more complex. Soon actual divisions of actual groups are not needed for the study. Step by step, slowly but surely, if his study is well managed, the pupil moves along to the idea of dividing into equal parts, of dividing tens as he has learned to divide ones, of dividing by tens, of making comparisons through divisions, and of using divisions, rendered into fractions and per cents, as relational ideas. Step by step, the pupil builds up his store of meaning of division by studying division.

The study of division extends at each step into the concrete. Social situations in which the idea of division appears in various guises are presented for the pupil to consider. They each confront the pupil with a question, commonly called a "problem," which requires the recognition of division for its proper answer. They are, at the outset, familiar situations, situations which do not need to be studied. Being familiar, they make the idea of division, which is their central characteristic, easy to recognize. Through the challenge which they make, they encourage repeated recognitions of division and thereby help to make division a much more familiar idea than it otherwise could possibly be. Through repeated recognitions, the idea finally becomes so familiar to the pupil that he can recognize it anywhere, even in situations which are relatively unfamiliar. At this point, step by step, the study of division may extend to new and less simple and more concretely complex situations, situations which need to be studied. Here may be noted a double movement of meaning or of the recognition of meaning. The pupil's familiarity with division helps him to grasp the meaning of the new situation, and the new situation extends his practice in studying division.

The central purpose of the pupil's exercises with social situations is to extend his method of studying division. The study of the situations is a means, not basically the end. The end sought is a fuller and richer meaning of division than would be possible without the extended means of study. Such fuller and richer meaning is not sup-

plied from and by the situations themselves, but from the entended study. The situations are important for the study, because they present division in a varied environment and in an ever-changing dress. Through the service they render, they are supplied meaning by the idea of division.

What has been said about the development of the meaning of division through the study of division applies to the development of the meaning of all the general ideas of arithmetic, whether they be addition and average or partition and per cent.

What the pupil can do with his developing meaning as he gets it has been indicated. Step by step, he is able to carry it, —first, his simple meaning and, later, his fuller and richer meaning,—first, to simple social situations of which division is an element and, later, to more and more complex social situations. He can do this step by step, as indicated, because when his meaning is scant and simple, his teacher selects as the objects of application simple situations that have few concrete qualities which may distract, and reserves for later application, when meaning is fuller and surer, the more complex situations possessing concrete qualities sometimes confusing in their abundance.

The meanings of numbers are gained likewise—through guided, systematic study of numbers. The pupil may study numbers through the thoughtful combinations and separations, in additions and multiplications and in subtractions and divisions, respectively, first, of actual groups and, later, of the ideas of groups in terms of the standard group of ten. He should perform these combinations and separations, if it is intended that he should study numbers, not as exercises of drill for skill, but as deliberate, thoughtful exercises to determine the sense of the answers that result. Skill will eventually be a by-product, but it is important only as an accessory after the fact. Applications are possible at every step of progress, but they are mere immediacies. The essential process is the process of studying numbers. As a consequence, the thoughtful teacher will guide the pupil's study of numbers in the manner indicated until the point of diminishing returns is about to be reached and then the thoughtful teacher will induct the pupil into methods of study that are more advanced. Such have already been indicated. They are the methods of studying numbers through comparisons and by means of relational ideas.

7. *Why not substitute the study of social situations?* Much study is a weariness of the flesh. Let us reduce it by substituting the study of social situations for the study of numbers and number processes. Is it not social situations that are important? Why not then make them the objects of study?

Social situations,—many that the pupil has already met and many that he as a maturing citizen must eventually meet,—are important. They have a double importance. In the first place, they develop the number meanings which the prepared pupil may bring to them or recognize in them. They furnish a kind of interest payment upon the meanings which are vested in them, and the amount of the interest is somewhat in proportion to the amount of the investment. They contribute to such number meanings by the enlarged and advanced opportunities they provide for furthering the study of numbers, number processes and number relations. They give the pupil the chance, if and when he is prepared to take it, of bringing his number thinking, such as he has developed, out into the complex and concrete realities of everyday life.

Secondly, social situations are important in their own right. The point need not be argued. Social situations are recognized as constituting the expanding social world into which the pupil is gradually moving. He cannot by-pass them. He may even be swallowed up in them. To move through them, however, he must be intelligent about them. In his private life he may gain satisfactions and in his public life he may meet his responsibilities in the

degree that he is able to assess and thereby bring into orderly relation the social situations that surround him.

Social situations have to be studied. Each is a complexity of many elements which appear in varying relations and proportions. Each is thus a fluid complexity assuming many particular forms. Social situations are hard to fit into a given mold; they are difficult to handle according to rules. To be understood, therefore, they must be taken apart and their inner operations subjected to critical view.

Social situations are apprehended and thus become objects of study in the degree that the number relations that characterize them are apprehended. Number is so much a part, so much the very essence, of social situations that an understanding of the one is a required preliminary to the understanding, even the contemplation, of the other. Social situations are *social* according to the *number* of persons that are involved; they are *situations* because the *number* of persons involved, the *number* of activities employed, the *number* of dollars expended, and the *numbers* of other factors that are present, appear in unpredictable, fluid, and evasive relations. They are situations to those who are able to sense the challenge they offer to bring their elements into balance.

Instead of social situations supplying meaning to number, exactly the reverse is true. Though they provide a medium in which number meaning may develop, they depend upon number for such meaning as they have or may finally come to have. For example, the fact of "sharing," which is a part of everyday experience, does not make division clear. It does not make itself clear. Instead, it is the idea of division, such as the pupil at the time may possess, that gives the fact of "sharing" the meaning it can have for him. Even the social implications and obligations of "sharing" remain nebulous to the pupil, however much his teacher may expatiate upon them, until his idea of division gives them an outline and a perspective. To illustrate further, consideration of installment buying does not supply meaning to percentage. Indeed, without the meaning of percentage consideration of anything other than the surface, objective features of installment buying is an impossibility. The pupil, l ke the credulous customer, may be told the cost of the conveniences of installment buy ng, but what the cost is as a relational amount is not self-revealing. The pupil must become as the wise customer by getting in his possession the idea of relational amounts if he is to be expected to gauge costs in their true proportions.

The study of social situations cannot substitute for the study of number and its processes and relations. It is naive to suggest that social situations can be studied in dissociation from the number relations that give them their character. It is thoughtless in the extreme to suggest that any such attempt will produce number ideas for the non-possessor. The *social arithmeticians* are correct in recognizing a movement of number meanings with respect to social situations. Their mistake is in trying to reverse the direction.

8. *Why have a double aim?* What has just been said applies, though in less degree, to those arithmeticians who would divide arithmetic into a *mathematical* phase and a social phase. They make the same mistake as do the *social arithmeticians*, though in less degree, of undertaking to substitute the study of social situations for the study of number and its processes and relations. They seek to foster the study of number, it is true; but they are inclined to turn their pupils quickly to the study of social situations. They have their reasons. In the first place, they desire to stimulate the study of number by the drive of a previously gained social motive. In the second place, they fear that an over-enthusiasm for the development of number meaning may lead to a neglect of social situations. Finally, they are concerned with the belief that the pursuit of number meaning is

bound to be narrowly conceived, "narrowly mathematical," and "abstract" or meaningless, without generous applications of social significance as an alleviating factor.

The least than can be said in criticism of the reasons that are commonly advanced in favor of a double aim for arithmetic is that they represent very poor timing. Impatience to pursue the *social aim* can result in nothing less than that both the *mathematical aim* and the *social aim*, whatever they may turn out to be, will be disturbed. The *mathematical aim* will suffer through partial neglect and, as earlier discussion has indicated, will be foreshortened by means which transcend were neglect. There will result an inadequate study of number and a consequent inadequate study of social situations. An over-weening fondness for the golden eggs bodes ill for the goose that lays them.

9. *Why not concentrate upon number meaning?* The arithmetic of the school suffers from two diseases or two forms of the same disease. (1) It aims at two things at once: it has the *social* aim and the *mathematical* aim. (2) It is considered by far too many teachers as a subject of secondary importance, as a subject having value only in the degree that it is *useful* in matters that are of primary importance. Too many teachers are apologetic about arithmetic and consume their energies seeking justification for the small amounts of attention they bring themselves to devote to it. As a consequence, the arithmetic of the school is often difficult to recognize as arithmetic. Its movement is frequently that of a pendulum back and forth from consideration of the tangible and superficial aspects of social situations, which is not arithmetic, to drill in the mechanics of processes, which is not arithmetic either.

What would happen to school arithmetic if teachers were to adopt a single aim, the single aim of fostering the development of the meanings of numbers, of number processes, and number relations? What would happen to school arithmetic if teachers were to turn their full energies upon the setting up of systematic studies by pupils of numbers, their processes and relations and upon the careful, patient guidance of pupils in pursuing such studies? What would happen to school arithmetic if teachers were to forsake their apologetic attitude toward arithmetic and begin treating it as a subject of major importance in its own right? What would happen?

Would number be presented for study in a vacuum? The answer that has been suggested is "No." Would the study of social situations be neglected? Again, the answer that has been suggested is "No." Instead, the pupil's study of number and its processes and relations might be systematized so that it would begin with the simplest of abstractions, and move along toward those that are less simple and more difficult to pull from their concrete surroundings, reaching along the way and finally toward the end those that the pupil should be trained to employ in important social situations. Once we prepare the pupil so he can approach the study of social situations in possession of number meanings already at least partially developed, we shall pave the way for a study of social situations that will be fruitful. It will be doubly fruitful. The pupil will develop the number of meanings he has already gained. He will become intelligent about the social situations he studies.

Would pursuit of the single aim produce a narrow course? It has been submitted herewith that pursuit of the single aim would result in a course that is both *socially* broad as well as *mathematically* ongoing.

◆ THE ART OF TEACHING ◆

A Technique for Solving Simple "Set Up" Equations in One Unknown

By MAY L. WILT

West Virginia University, Morgantown, W. Va.

NO LONGER can teachers of mathematics be satisfied with the type of teaching that produces uncertain results. The first year of algebra is a good starting point for developing teaching techniques. Most children, who have had one year of algebra, can solve simple "set up" equations in one unknown if they are directed "what to do next." They should be able to know "what to do next." Very bright children find out for themselves, but the average and slower ability groups must be helped to develop a technique that will produce certainty and accuracy in results.

A method in use in the University High School at West Virginia University is giving excellent results. It is in keeping with the recent trends toward the elimination of transposition in teaching simple equations.

It consists of six separate steps:

1. The "intuitive period."
2. The "question period."
3. The "how equations are built up and taken apart period."
4. The "unknown in both members period."
5. The "accumulative equation period."
6. The "controlling period."

1. During the "intuitive period" the child learns that:

a. Any number, except zero, divided by itself equals one.

b. $+n$ (any number) plus $(-n)$ equals zero. This procedure is an excellent method for eliminating any term that is not desired.

c. The "equation idea" provides unlimited opportunities for representing relationships in mathematics, the sciences, and in many every day procedures.

d. The appearance of an equation may be changed, without relating its fundamental truth, through the operation of the four basic axioms of addition, subtraction, multiplication, and division.

e. The equation asks a question that is answerable.

2. The "question period"—during which the child learns to ask the question indicated in a simple one operation equation, to estimate a sensible answer, and to check for accuracy.

The following types of equations are used for introductory purposes.

$$x+3=12 \qquad 3m=24$$

$$a-4=7 \qquad \frac{y}{3}=10$$

If the equation is expressed as a question the wording would be as follows:

To what number can 3 be added so that the sum equals 12?

From what number can 4 be subtracted so that the remainder equals 7?

By what number must 3 be multiplied so that the product equals 24?

What number, divided by 3, gives a quotient of 10?

In each case the pupil determines a sensible answer and makes sure that it will check. Sufficient experience is necessary to make the child sure of himself. Errors

should be detected, analyzed, and clarified.

From this step the question naturally follows, "Which of the four axioms explains what happened in each case?" The student should reason out that the following has taken place:

$$\begin{array}{rl} x+3= & 12 \\ -3= & -3 \\ \hline x= & 9 \end{array} \qquad \frac{3m}{3}=\frac{24}{3} \quad m=8$$

$$\begin{array}{rl} a-4= & 7 \\ +4= & +4 \\ \hline a= & 11 \end{array} \qquad 3\times\frac{y}{3}=10\times 3 \quad y=30$$

When this period is understood he should be ready for such variations as:

$$a+\tfrac{1}{2}=10\tfrac{1}{2} \qquad 3\%y=124$$

$$6x+2x=48 \qquad \frac{c}{2}=5.4$$

$$m-.5=4.6 \qquad \tfrac{1}{4}c=12$$

$$.4x=9.6 \qquad ax=b$$

$$x+c=10 \qquad \frac{y}{a}=m$$

$$x-m=n$$

3. The "how equations are built up and taken apart period" grows out of building identities. Take any number, such as 12, and build a series of identities. Then explain how they may be taken apart.

$$4\times 12-6=42$$

b.u.	t.d.
×4	+6
−6	÷4

$$\frac{12}{3}+20=24$$

b.u.	t.d.
÷ 3	−20
+20	× 3

$$24=\frac{5\times 12}{2}-6$$

b.u.	t.d.
×5	+6
÷2	×2
−6	÷5

$$\frac{4\times 12}{3}-10=6$$

b.u.	t.d.
× 4	+10
÷ 3	× 3
−10	÷ 4

The pupil is now ready to understand that these identities may be changed to simple equations in which the 12 is missing and its location occurs in only one member as:

$$4x-6=42$$

$$\frac{x}{3}+20=24$$

$$\frac{4x}{3}-10=6$$

$$24=\frac{5x}{2}-6$$

The pupil then asks the question expressed by the equation. He should indicate, after each equation how it is built up and how it must be solved. The exercises should be arranged in the following manner, solved, and then checked:

$$\begin{array}{rl} 4x-6= & 42 \\ +6= & +6 \\ \hline \frac{4x}{4}= & \frac{48}{4} \end{array}$$

$$x=12$$

b.u.	t.d.
×4	+6
−6	÷4

$$\begin{array}{rl} \frac{x}{3}+20= & 24 \\ -20= & -20 \\ \hline 3x\,\frac{x}{3}= & 4\times 3 \end{array}$$

$$x=12$$

b.u.	t.d.
÷ 3	−20
+20	× 3

$$\begin{array}{rl} \frac{4x}{3}-10= & 6 \\ +10= & +10 \\ \hline 3x\,\frac{4x}{3}= & 16\times 3 \end{array}$$

b.u.	t.d.
× 4	+10
÷ 3	× 3
−10	÷ 4

$$\frac{4x}{4}=\frac{48}{4}$$

$$x=12$$

$$24=\frac{5x}{2}-6$$

$$+6=+6$$

$$2\times30=\frac{5x}{2}\times2$$

$$\frac{60}{5}=\frac{5x}{5}$$

$$12=x$$

or

$$x=12$$

b.u.	t.d.
×5	+6
÷2	×2
−6	÷5

As the pupil advances he should be able to perform most of the steps, in the solution of the equation, in his head. This, of course, is the ultimate goal, but should not be undertaken until there is complete understanding. The plan of solution should persist throughout the whole ninth year. The student should be ready to plan, solve, and check any simple equation in one unknown in which the unknown is located in only one member. He should then work on literal equations of the following types and note similarity to the types already studed:

$$x+a=b+10$$

$$y-4=c \qquad cy+d=m+n$$

$$\frac{ax}{c}+b=d$$

4. The "unknown located in each member period" should require two additional steps:

a. Simplifying each member so as to put the equation question in a simpler form.
b. Changing the resulting equation to one in which the unknown occurs in only one member.

These two steps may be illustrated through the solution of the following equation:

$$12x-20-6x+4=80-2x-36+6x$$

$$6x-16=44+4x$$

$$+16=+16$$

$$6x=60+4x$$

$$-4x=\quad-4x$$

$$2x=60$$

$$x=30$$

b.u.	t.d.
×2	÷2

The answer should be checked. Equations, involving fractions written in decimal form, need an extra step. Experimenting with the four different axioms and the proposed values should help the student find out that the following procedure is necessary:

$$.5x+.06=.3x+.82$$

$$100(.5x+.06)=100(.3x+.82)$$

$$50x+6=3x+82$$

He is prepared to complete the solution. Additional practice will include one place decimals, two place decimals, and mixtures of one and two place decimals.

5. "Accumulative equations" necessitate a greater knowledge of the four fundamental operations including monomials and polynomials, as well as the elementary types of factoring.

In the following types the pupil should know that the expressed multiplication should be performed before the exercise is ready for analysis for the next step.

$$6(x+3)=4(x+12)$$

$$(y+3)(y+10)-162=(y-6)(y+10)$$

Various errors will develop and should be studied and clarified.

If common or algebraic fractions occur, the pupil must be directed to take the following steps:

a. Find the least common denominator.
b. Determine the most plausible axiom to use and the expression that will reduce each denominator to one.
c. Perform any indicated multiplication remaining, collect, change to an

equation with the unknown in only one member, solve, and check.

The following equations illustrate the types that will be used:

$$\frac{4x}{x+2}-\frac{4}{x-2}=\frac{2(5x+2)}{x^2-4}$$

$$\frac{x+3}{x-8}-\frac{2x^2-2}{x^2-7x-8}=\frac{5-x}{x+1}$$

The pupil should then be in command of all types of simple "set up" equations, involving one unknown, found in any ninth grade course, in plane geometry, and in general science.

6. "Controlling equations" is a resultant of studying simple equations as a whole, understanding the common types and methods of procedure, and in devoting the greater part of the practice period to mixed groups containing all types. The child then will be faced with the necessity of thinking. Too frequently the practice period in algebra consists of solving long lists of similar equations merely by initiation.

Why children make errors and how to overcome them becomes the real opportunity for study and the solemn obligation of the teacher of mathematics. Many times the child's reactions are correct, according to his understanding, and the teacher fails to help him make needed adjustments.

"Set up" equations represent but one phase of the simple equation and yet they are absolutely essential before the second phase can be undertaken. Too frequently difficulties become accumulative and the child becomes hopelessly lost in mathematics.

The second phase, the "story" or "verbal equation" is a study within itself and requires the development of a technique that produces certainty and accuracy equivalent to that developed for the solution of "set up" equations. Because it takes real thought and preparation, too many mathematics teachers overemphasize manipulative algebra and underemphasize the thought provoking phases.

EDITORIALS

Allan Congdon, professor of mathematics and education at the College of Education at the University of Nebraska retired from active service in May 1945. Professor Congdon has been one of the outstanding teachers at the University since 1919, when he was appointed Associate-Professor. In 1930 he received the title of professor of mathematics and in 1934 of professor of education.

Professor Congdon was born in Jamestown, N. Y. on June 9, 1876. He taught in a rural school beginning in 1893 at the age of 17. He entered the University of Nebraska in the fall of 1894. He received an appointment as "approved tutor in mathematics" in 1895, and his bachelor's degree there in 1899. In 1923 he received his master's degree from the University of Nebraska. He received the Ph.D. degree from Columbia University in 1929. Professor Congdon taught every year (at least part time) from 1895 until 1945. Prior to his appointment at the university, Professor Congdon taught in Lincoln and Omaha, was principal of Fremont high school and superintendent at Wahoo. While completing his doctor's work, he was instructor at the City College of New York and Teachers College, Columbia University.

Active in many national organizations, Dr. Congdon has held various offices, including national treasurer of Phi Delta Kappa since 1925 and vice president and member of the board of directors of the National Council of Teachers of Mathematics. He is also a past-president and secretary of the Nebraska chapter of Phi Beta Kappa, and a member of the National Education association, American Association of University Professors, American Interprofessional Institute and Mathematics Association of America.

As chairman of the important university assignment committee and chairman of the course of study committee for Teachers College, Dr. Congdon served for some time.

Among his publications is "Training in Mathematics Essential for College Success." He is also co-author of "Remedial Arithmetic," published in 1937.

Dr. Congdon died early Sunday morning at his home in Lincoln at the age of 69 after this editorial had gone to the printer. His passing is a great loss to the National Council of Teachers of Mathematics whom he served so long and so well. He was a truly great teacher and friend.

W. D. R.

Annual Meeting of the National Council of Teachers of Mathematics

As announced in the December issue of The Mathematics Teacher, the next annual meeting of the National Council of Teachers of Mathematics will be held at the Hotel Statler in Cleveland, Ohio, on Friday and Saturday, February 22nd and 23rd, 1946. A meeting of the Board of Directors of the Council will be held at 8 p.m. on Thursday, February 21st, at the Hotel Statler. The final program of the meeting will appear in the February issue of the Teacher.

There will be four sectional meetings on Friday as follows:

1. Mathematics in the Elementary School
2. Mathematics for Grades Ten to Fourteen
3. Mathematics in the Junior High School
4. The Training of Teachers of Mathematics and one general meeting devoted to the work of the Commission on Post War Plans. A Discussion Banquet will be held on Friday evening. Saturday morning will be devoted to a radio demonstration and a general session on multi-sensory aids. Let us all try to be there.—W. D. R.

◆ IN OTHER PERIODICALS ◆

By NATHAN LAZAR
Midwood High School, Brooklyn 10, New York

American Mathematical Monthly
October 1945, Vol. 52, No. 8

1. Widder, D. V., "What is the Laplace Transform?", pp. 419–425.
2. De Cicco, John, "Circle-to-Line Transformations," pp. 425–433.
3. Weinstein, A., and Pounder, J. R., "An Electromagnetic Analogy in Mechanics," pp. 434–438.
4. Weiss, Marie J., "Discussions and Notes," (Ed.), pp. 439–443.
   (a) Kaplansky, Irving, "Minimal Tangents"
   (b) Sternberg, W. J., "On Polynomials with Multiple Roots"
   (c) Grossman, H. D., and Kramer, David, "A New Match-Game"
5. Frame, J. S., (Ed.), "Clubs and Allied Activities," pp. 443–447.
6. Evans, H. P., (Ed.) "Recent Publications," pp. 448–456.
7. Dunkel, Otto, Frink, Orrin, Jr., Eves, Howard, (Ed.), "Problems and Solutions," pp. 457–467.
8. Jones, B. W., (Ed.), "News and Notices," pp. 468–469.
9. Newsom, C. V., (Ed.), "General Information: Mathematics in Engineering Graduate Study; Predoctoral Fellowships in the Natural Sciences; New NROTC; Postwar Educational Services for Veterans and Service Personnel," pp. 470–475.

November 1945, Vol. 52, No. 9

1. Richmond, D. E., "Elementary Evaluation of Laplace Transforms," pp. 481–487.
2. Farnell, A. B., "Limits for the Field of Values of a Matrix," pp. 488–493.
3. Goodman, A. W., and Goodman, R. E., "A Circle Covering Theorem," pp. 494–498.
4. Brand, Louis, "The Lagrange Identity as a Unifying Principle," pp. 499–502.
5. Finan, E. J., "Magic Rectangles Modulo p," pp. 502–506.
6. Olmsted, J. M. H., "Rational Values of Trigonometric Functions," pp. 507–508.
7. Thébault, Victor, "The Area of a Triangle as a Function of the Sides," pp. 508–509.
8. Frame, J. S., (Ed.), "Clubs and Allied Activities," pp. 510–514.
9. Evans, H. P., (Ed.), "Recent Publications," pp. 514–515.
10. Dunkel, Otto, Frink, Orrin, Jr., and Eves, Howard, (Ed.), "Problems and Solutions," pp. 516–533.
11. Jones, B. W., (Ed.), "News Notices," pp. 533–535.
12. Newsom, C. V., (Ed.), "General Information: The Cooperative Committee on Science Teaching; A Course for Prospective Teachers in Ohio; Academic Interests of Discharged Veterans; Teachers for the Veteran Rehabilitation Program; Important Legislation before Congress," pp. 536–540.
13. The Mathematical Association of America, "New Members," pp. 541–542.

Bulletin of the Association of the Teachers of Mathematics of the City of New York
May 1945, Vol. 1, No. 2

1. Peters, Max, "Editorial," pp. 1–2.
2. Welkowitz, Samuel, "A Case of 'Integral Continuity'," pp. 3–8.
3. Altwerger, Samuel, "Articulation Needed," pp. 9–10.
4. Grossman, George, "Using the Formula for Enrichment," pp. 11–14.
5. Charosh, Mannis, (Editor), "The Problem Page," pp. 15–18.
6. "Book Reviews," pp. 18–20.
7. "On Imaginaries," p. 20.

The Duodecimal Bulletin
October 1945, Vol. 1, No. 3

1. "An Important Reprint," pp. 3–7.
2. United Nations Standards Coordinating Committee, pp. 7–8.
3. Andrews, F. Emerson, "About Decimal-Form Fractions," pp. 9–10.
4. "The Opposed Principles" (An Editorial), pp. 10–13.
5. Seelbach, Lewis Carl, "Consovocalic," pp. 14–16.
6. Ingalls, W. R., "Systems of Weights and Measures," (Review), pp. 16–17.
7. "Duodecimal Bibliography," pp. 17–21.
8. "Growing Teacher Interest," pp. 21–22.
9. Lloyd, Mary, (Ed.), "Dosenile Department," pp. 22–23.
10. "Official Announcements," pp. 23–24.
11. "The Mail Bag," pp. 24–31.

National Mathematics Magazine
May 1945, Vol. 19, No. 8

1. James, Glenn, "Evaluation of Real Roots by Means of Lower Degree Equations," pp. 375–384.
2. Grossman, Howard D., "Applications of an Operator to Algebra and to Number Theory, with Comments on the Tarry-Escott Problem," pp. 385–390.
3. Poor, Vincent C., "On the Motion of a Rigid Body," pp. 391–394.
4. Goodrich, Merton Taylor, "A Systematic Method of Finding Pythagorean Numbers," pp. 395–397.
5. Lu, Chin-Shih, "Some New Properties of the Triangle," pp. 398–405.
6. Schaaf, William L., "Perversion of Purpose in Elementary Mathematical Education," pp. 406–413.
7. Philip, Maxmilian, "A Note on Simple Interest," pp. 414–417.
8. Wiggin, Evelyn, "The Value of Mathematics in a Liberal Education," p. 418.

9. Starke, E. P., and Court, N. A., (Ed.), "Problem Department," pp. 419–427.
10. Simmons, H. A., and Smith, P. K., (Ed.), "Bibliography and Reviews," pp. 428–430.
11. Index to Volume 19 (October, 1944 to May, 1945), pp. 432–436.

October 1945, Vol. 20, No. 1

1. Sanders, S. T., "Mathematical Achievement and the Specific Problem," p. 2.
2. Bell, E. T., "A Representation of Certain Integer Powers," pp. 3–4.
3. Wade, T. L., "Tensor Algebra and Invariants, II," pp. 5–10.
4. Sleight, Norma, "Pertinent Historical Material for a Slide Rule Course," pp. 11–20.
5. Keal, Harry M., "Whither Mathematics?", pp. 21–28.
6. Boyer, Carl B., "Fermat's Integration of X^n," pp. 29–32.
7. Buck, R. C., "Multiple Integration," p. 33.
8. Starke, E. P., and Court, N. A., (Ed.), "Problem Department," pp. 35–42.
9. Simmons, H. A., and Smith, P. K., (Ed.), "Bibliography and Rzviews," pp. 43–48.

School Science and Mathematics

November 1945, Vol. 45, No. 8

1. Kinney, Jacob M., "The Function Concept in the Solution of Elementary Algebra," pp. 693–703.
2. Breslich, E. R., "Mastery of the Fundamentals of High School Mathematics: A Graduation Requirement," pp. 743–756.
3. Goins, Jr., William F., "A Homemade Transit," pp. 766–768.
4. Nyberg, Joseph A., "Notes from a Mathematics Classroom," pp. 770–773.
5. Jamison, G. H., (Ed.), "Problem Department," pp. 774–779.
6. "Books and Pamphlets Received," pp. 779–780.
7. "Book Reviews," pp. 780–783.
8. "Bill of Rights of Teachers of Secondary Mathematics," pp. 784–785.

Scripta Mathematica

March 1945, Vol. 11, No. 1. (completed)

6. Whitlock, Jr., W. P., "Pythagorean Triangles with a Given Difference or Sum of Sides," pp. 75–81.
7. Quotations, pp. 20, 96.
8. Curiosa, p. 81.
9. Stewart, Jr., Norman, "The Anatomy of Magic Squares," pp. 85–88.
10. Anema, Andrew S., "Franklin Magic Squares," pp. 88–96.
11. Landes, Leon, "On Equiareal Pythagorean Triangles," pp. 97–99.
12. Juzuk, Dov, "On the Converse of Fermat's Theorem," p. 100.
13. "Selected Items from the Columbia University Collection of Mathematical Models," (Photograph), facing p. 72. (Selection, arrangement and lighting by Rutherford Boyd).

Proposed Amendments of the By-laws of the National Council of Teachers of Mathematics, Incorporated

A. Article II, 2, To read: Membership in the Council is effected by the payment of an annual membership fee determined by the Board of Directors. This fee includes one year's subscription to "THE MATHEMATICS TEACHER."

B. Article II, 3,—Delete.

C. Article III, 1, To read: The control and management of the affairs and funds of the Council shall be vested in a Board of Directors who shall be members of the Council. The Board of Directors shall consist of fifteen (15) voting members: the president, three vice-presidents, secretary-treasurer, editor, and nine directors. The most recent past-president shall be an ex-officio but nonvoting member of the Board.

D. Article III, 2,—Delete.

E. Article III, 3, Change to 2 to read: The President shall be elected by the members of the Council biennially for a term of two years and shall not be eligible for a second successive term. The three Vice-Presidents shall be elected by the members of the Council biennially for a term of two years and shall not be eligible for election to a second successive term. Their terms shall run concurrently with the President's term. Three members of the Board shall be elected by the members of the Council annually for a term of three years as provided in (new) 3. They shall be eligible for re-election but not for more than two consecutive terms. The Secretary-Treasurer and the Editor shall be appointed by the Board for a term of three years. They shall be eligible for re-appointment. Their terms shall run concurrently. The Committee on Official Journal shall consist of the Editor and three Associate Editors. The Associate Editors shall be appointed by the Board subject to the approval of the Editor, for a term of three years, their terms running concurrently with the Editor's term. They shall be eligible for re-appointment but not for more than two consecutive terms. All members of the Board of Directors shall hold over until their respective successors are elected or appointed and qualify.

F. Article III, (new) 3, to read: The nine (9) members of the Board, other than the officers, shall be elected by the members of the Council according to a geographic plan as determined by the fifteen voting members of the Board.

G. Article III, 4, The first two sentences to remain the same, then to read: The Finance Committee shall consist of the President and the Secretary-Treasurer, who shall have oversight of expenditures under the direction of the Board. Both shall sign all checks drawn by the treasurer. The Board is authorized to create and fill additional offices and committees.

H. Article III, 6, Amend line 2 by changing "five (5) members" to "five (5) voting members."

I. Article III, 9, Change first word to "Any; and in third line change to "determined alphabetically."

J. Article VI, 1—Delete the last five words.

K. Article VII,—Delete.

L. Article VIII, 1,—Delete the first sentence and the first word of the second sentence.

December, 1945

Respectfully submitted,

1. EDWIN W. SCHREIBER, Chairman
2. MARTHA HILDEBRANDT, GEORGE E. HAWKINS
3. A. R. CONGDON, H. W. CHARLESWORTH
4. H. C. CHRISTOFFERSON (overseas)

Committee on Reorganization of By-Laws.

NEW BOOKS

1. Almstead, Francis E., Davis, Kirke E., and Stone, George K., *Radio, Fundamental Principles and Practices.* McGraw-Hill Book Co., Inc., New York, 1944. 208 pp. $1.80.
2. Apalategui, J. J., and Adams, L. J., *Aircraft Analytic Geometry.* (Applied to Engineering, Lofting and Tooling), McGraw-Hill Book Co., Inc., New York. 1944. 281 pp. $3.00.
3. Atkinson, Carroll, *Pro and Con of the Ph.D.* Meador Publishing Co., Boston. 1945. 172 pp. $2.00.
4. Atkinson, Carroll, *True Confessions of a Ph.D. and Recommendations for Reform.* Meador Publishing Co., Boston. 1945.
5. Betz, William, *Everyday Junior Mathematics.* (books 1 and 2) Ginn and Company, Boston. 1944. Book 1, 400 pp. $1.12. Book 2, 480 pp. $1.28.
6. Clark, John R., Otis, Arthur S., and Clark, Caroline Hatton, *My First Number Book.* World Book Company, New York. 1945. 80 pp.
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9. Einstein, Albert, *The Meaning of Relativity.* Princeton University Press, Princeton. 1945. 132 pp. $2.00.
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13. Littlewood, J. E., *Lectures on the Theory of Functions.* Oxford University Press, London. 1944. 243 pp. $5.50.
14. Mason, Louis T., *Physics Made Easy.* (revised edition) School Science Press, Buffalo. 1943. 394 pp.
15. Middlemiss, Ross R., *Analytic Geometry.* (first edition) McGraw-Hill Book Co., Inc., New York. 1945. 306 pp. $2.75.
16. Morgan, Frank M., *Plane and Spherical Trigonometry.* American Book Co., New York. 1945. 247 pp. $2.50.
17. Neville, Eric Harold, *Jacobian Elliptic Functions.* Oxford University Press, London. 1944. 331 pp.
18. Nyberg, Joseph A., *Fundamentals of Algebra.* (second book) American Book Co., New York. 1945. 405 pp. $1.60.
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